APPLICATION OF THE WAVELET TRANSFORM TO FILTERING
AIRBORNE LASER SCANNING DATA

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Abstract
In the recent years the meaning of the airborne laser scanning as the technique for obtaining precise data of terrain has grown. One of the priority results, which can be derived from ALS data is Digital Terrain Model. Filtering is the first step in processing the airborne laser scanning data. This process is based on elimination of all points which were reflected by objects which do not represent the physical surface of the ground. It means that in the 3D point cloud all construction, vegetation, engineering objects should be eliminated, when naturally formed elements of land should remain intact. The problem which may come across during the filtering process is huge datasets of airborne laser scanning. This makes the need to seek fast and effective filtering algorithms. This paper presents a method of the filtering based on discrete fast wavelet transform. The studies and practical experiences show that the method is very effective and allows the filtering of one million points in the time of 3.4 seconds. This makes the developed algorithm fulfills its role perfectly and brings intended results.

Keywords
Discrete wavelet transform, Airborne laser Scanning, filtering, ScALARS

1. Wavelets

Wavelets are mathematical expressions that satisfy special requirements. The name wavelet suggest that these functions integrate to zero, waving above and below the x-axis. Like sines and cosines in Fourier analysis, wavelets are used as basic functions in representing other functions (especially non-continuous functions). The basic difference between Fourier and wavelet transform is the number of parameters. Fourier basic functions are localized in frequency but not in time. Small frequency changes in the Fourier transform produce changes everywhere in the time domain. Wavelets are local in both frequency and time, what ensures a very great advantage of wavelet transform [13].

There are many sorts of wavelets, so one can select the kind of function for the particular application. One of the simplest is Morlet’s wavelet, which has got a lot in common with cosines function (used in Fourier transform). The “basic” cosines is just multiplied by a coefficient that makes the function extinct for the arguments far from zero.

![Example of simple wavelets](image.png)

The primary advantage of the fast wavelet transform (FWT) is computational complexity, which in the worse case is $O(n)$, when the computational complexity of fast Fourier transform (FFT) is $O(n\log_2 n)$. It makes that decomposition of the ALS signal can processed approximately $\log_2 n$ faster even than using the FFT [13].

Furthermore wavelet transform is very well localized both in time and frequency domain. When the Fourier transform decomposes 1-dimensional signal in domain of time into 1-dimensional domain of frequency, the wavelet

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transform decomposes signal in the same based domain into the 2-dimensional domain of time and frequency. This is possible through two parameters – one responsible for scale, and the second responsible for dilatation. This property makes the wavelet transform useful for analysis both in time and frequency domain.

The continuous wavelet transform is based on equation [3]:

\[ s_{\psi}(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} s(t) \psi\left(\frac{t-b}{a}\right) dt \]

(1)

The coefficient \(a\) is responsible for both amplitude and scale of the wavelet, when the coefficient \(b\) responds for dilatation. The element \(s(t)\) means the value of signal in the time \(t\), and \(\psi\) means the value of wavelet function in the particular point. Above equation can be used directly for the continuous wavelet transform or may be reconstructed into the discrete form.

1.1 Wavelet decomposition

Many data operations can be done by processing the corresponding wavelet coefficients. The figure below shows the way of decomposition of the signal. The process of decomposition of the signal may consist of several steps. In every step (level of transformation) one finds values of transform coefficient \(s(a,b)\) of the scaling function as well as the wavelet function, where \(a\) is constant on the entire level. The set of orthogonal function – wavelet (low-pass filter) and scaling function (high-pass filter) is called the quadrature mirror filters. These coefficients enable reconstruction of the approximated signal through inverse wavelet transform. The approximated signal on the first level of the decomposition has got twice worse resolution than the original one. The residuals between the signal on the higher level and the approximation are called details (obtained by wavelet function). Details on the particular level of the transformation have got the same resolution as the signal on the higher level (twice better than the approximation on the same level). In the next step (level) of transformation, one takes under consideration only the approximation of the signal (due to scaling function). Details show the residuals and are left unchanged. To reconstruct original signal one has to take the approximation on the highest level and values of details on each level of transformation.

Graph 1 Decomposition of the signal using wavelet transform according to Mallat’s schema [3]

2 FILTERING OF AIRBORNE LASER SCANNING DATA

Airborne Laser Scanning (ALS) is a very efficient tool of remote sensing for creating Digital Terrain Model as the digital representation of ground surface topography. In this method active remote sensing instrument transmits electromagnetic radiation and measures the anti-radiation that is scattered back to the receiver of the ALS system after
interacting of the various surface. After a single ALS sensing campaign a huge amount of point dataset is obtained.
Filtering is the first step in processing the airborne laser scanning data. This process is based on elimination of all points which were reflected by objects which do not represent the physical surface of the ground. It means that in the 3D point cloud all construction, vegetation, engineering objects should be eliminated, when naturally formed elements of land should remain intact.

**Fig. 2  The trace the laser beam of ScaLARS system**

The process of filtering ALS data can be performed in time domain or frequency domain. Among the first type of filtering methods stand out among others:
- Methods based on robust linear prediction of the asymmetric suppression [8]
- Methods, consisting of an iterative bringing off a surface (TIN) to the measured data [1]
- Methods using mathematical morphology operators (criterion inheritance of ground) [11]
- Methods using cluster analysis [10]
- Methods based on the minimization of surface energy which depends on the slope (decrease) [7] and a generalized version [4]

Methods of filtering in the frequency domain include algorithms based on Fourier transform as well as wavelet transform. Deserve special attention include:
- Methods based on Fast Fourier Transform (FFT) and low-pass filters with finite impulse response (FIR) [9],
- Methods for the extraction of specific parts of the field using a field-length wavelets transformation and supervised classification [12]
- Methods to eliminate large-area variations based on the high-pass filter of discrete wavelet transform [2]
- Methods of extinction a certain level of detail wavelet decomposition [5].

Taking into consideration all the requirements for data filtering methods, in particular the computational complexity of the algorithm, minimizing errors, and the universality according to land cover, a algorithm has been developed for filtering the ALS data. This method allows for rapid and effective separation of the points representing the land surface from the points recorded as objects of land cover (trees, buildings, engineering structures), while leaving the essential elements of field structures, such as river embankments, ditches, hills and slopes. Due to adapt to the global path of the terrain, the method can be used both in the flat, and in the hilly area. In contrast to most algorithms working in the field coordinates, the proposed method is based on the original data, along the trace of the laser frequency in the wavelet field. Filtering is so one-dimensional character, without the need of converting the coordinates to a regular grid (GRID).
2.1 Test data

In this paper the data of the airborne laser scanning of Widawa river valley near Wroclaw was analyzed, where a prototype scanner – ScaLARS was used [3]. The project involved scanning 20 km of the Widawa river estuary with a with range of about 2 km. The mean error of calibration in reference to control areas carried out 0.3 m along the flight direction and across the flight direction, as well as 0.1 m in the vertical direction. Approximately 150 million points were registered with an average resolution 3pts/m² [4]

![Fig. 3 ALS data derived from the same area: before filtering (left) and after filtering (right)](image)

2.2 ALS data filtering based on low-pass filter of discrete wavelet transform

The developed algorithm consist of two-steps filtering, and provides the process of filtering in the domain of wavelet frequency. In this process, high frequencies of the signal, which can be thought as the terrain profiles, correspond to surface objects. Low frequencies are basically responsible for the surface of the ground.

In the first step of filtering process, based on low-pass filter of discrete wavelet transform, the identification of all points, which are far from the approximation surface, is made. Then the depreciation of the height of points is done. Furthermore the construction of the final approximation surface, which is intact by the influence of terrain objects, finishes the process of filtering. The algorithm works properly in the flat area, as well as in mountain terrains.

The process of filtering can be described as follow:
1) Wavelet decomposition on the $n$ level,
2) Hard threshold for all the details at the maximum level,
3) Synthesis of the signal back to the time domain,
4) The imposition of the mask data for the outliers above the surface,
5) Reduction the height of data with a mask,
6) Wavelet decomposition on the $n$ level,
7) Hard threshold for all the details at the maximum level,
8) The synthesis of the signal back to the time domain - to obtain wavelet approximation of the terrain profile,
9) Filtering - the analysis of each point of derogation from the surface approximation.
2.3 The verification of the algorithm

The algorithm was implemented in MATLAB environment using additional wavelet toolboxes. The method has been tested on the real data of the airborne laser scanning from the “Widawa Valley River” campaign in 2005. The result of every test data set were compared with the model data, considered as an error-free. It made possible to examine the accuracy and performance computing of the method. The results of filtering are appropriate. The accuracy of algorithm was estimated on 95%, with the possibility of filtering of 1 million of points in the time of 3.4 seconds.

Fig. 4  Wavelet db4 at the first (yellow) and second step (red) of decomposition ALS data.

Fig. 5  The result of filtering in the sloping terrain. As shown in figure, the wavelet approximation of the surface is adapted to the ground: vegetation is eliminated, while slope remains intact.
3 CONCLUSION

In this paper the possibility of application of the discrete wavelet transform to fast filtering of ALS data is described. For this purpose the algorithm of two-steps filtering has been developed. This method makes the process of filtering in the domain of wavelet frequency, where high frequencies of the signal (which can be thought as the terrain profiles) correspond to surface objects. Low frequencies are basically responsible for the physical surface of the ground. The algorithm works properly in the flat area, as well as in hilly and mountain terrains. The results of filtering are satisfactory. The accuracy of algorithm was estimated on 95%, with the possibility of filtering of 1 million of points in the time of 3.4 seconds.

Fig. 6 The height of ALS points (coded by grey scale – left) and the result of filtering on the orthofoto (right)

Literature


Reviewer

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