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WARSAW UNIVERSITY OF TECHNOLOGY
FACULTY OF GEODESY AND CARTOGRAPHY
DEPARTMENT OF GEODESY AND GEODETIC ASTRONOMY

MATHEMATIC – PHYSIC MODELS OF HORIZONTAL REFRACTION IN ENGINEERING AND INDUSTRIAL MEASUREMENTS OF THE HIGHEST PRECISION

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1. INTRODUCTION

Experimental and theoretical researches on limiting or elimination of interaction of target beam refraction on results of precise measurements of horizontal angles in industrial network are currently carried out around the world in the following, vital directions:

- **A) defining refraction effects on theoretical models;**
- **B) optimization of points of location of networks according to buildings surrounding them;**
- **C) experimental researches in research centers at various times of the day, year and with changing meteo-parameters;**
- **D) defining of partial refraction angles based on angles of light beam dispersion by means of so called dispersometers – instruments with high precision of optics and photo-detection receiving systems and automatic transformation of observation results.**

The work presents the first of research directions. Formulas for defining the effects of refraction for targeting, taking into consideration their location in the surroundings of typical buildings in industrial areas which emit strong temperature fields.

2. THEORETIC BASE FOR DEFINING FRAGMENTARY ANGLES OF HORIZONTAL REFRACTION

Fragmentary angles of refraction for temperature field along the *whole target length* (fig. 1) is defined by means of a definition formula (*Moritz H. 1962, Ostrovskij A.Ł 1992*)

$$\delta_A = -\delta_B = -\frac{1}{L}\int_0^L \frac{1}{n}\frac{\partial n}{\partial y} x \cdot dx = -\frac{1}{2}\sum_0^A d_i \cdot \nabla_y n_i = -\frac{1}{2} L \cdot \nabla_y^{sr} n_i \qquad \textbf{(1)}$$

on condition that refraction curve is three dimension conform projection of a surface onto a plane. This assumption allows elaboration of a uniform theory for all possible types of occurrences of refraction in geodesy. It means that in place of relatively complicated curves of a way of light geodetic lines appear. Taking into consideration the

speed of light and discreet modeling geodesy appears in the form of a circular curve. (*Moritz H. 1962*).

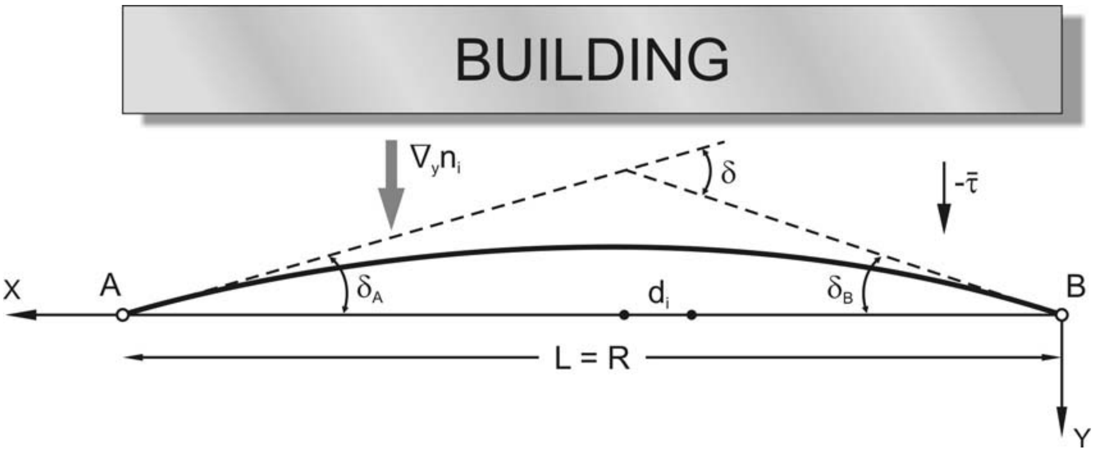

Fig. 1. Influence of refraction field along the whole target length.

Increment $\nabla_y n_i$ which defines a gradient of a factor of air refraction is determined on the base of measured meteorological elements. It has a vital meaning in creating geometry of refraction curves and at the same time by calculating values of refraction effects.

$$\frac{\partial n}{\partial y} = \nabla_y n = \frac{\partial n}{\partial T} \cdot \frac{\partial T}{\partial y} = -(n_0 - 1)\frac{T_0}{T^2} \cdot \frac{p}{p_0} \cdot \frac{\partial T}{\partial y} \quad \textbf{(2)}$$

For values of :
T = 293,16 K (average air temperature) T_0 = 273,16 K
p = 990 hPa (average atmospheric pressure) p_0 = 1013,25 hPa
n_0 = 1,00030222 (factor of refraction of air for normal conditions (*Bryś H. 1985*)
we receive: $\nabla_y n = -0{,}94 \cdot 10^{-6} \cdot \bar{\tau}$, where: $\bar{\tau} = \frac{\Delta T}{\Delta y_{\Delta y - 0}}$ gradient of air temperature defined by electronic meter of a difference of temperature parallel to aiming line) (*Bryś H. 1994*).

3. INFLUENCE OF REFRACTION AT THE END SECTOR OF AIMING LINE

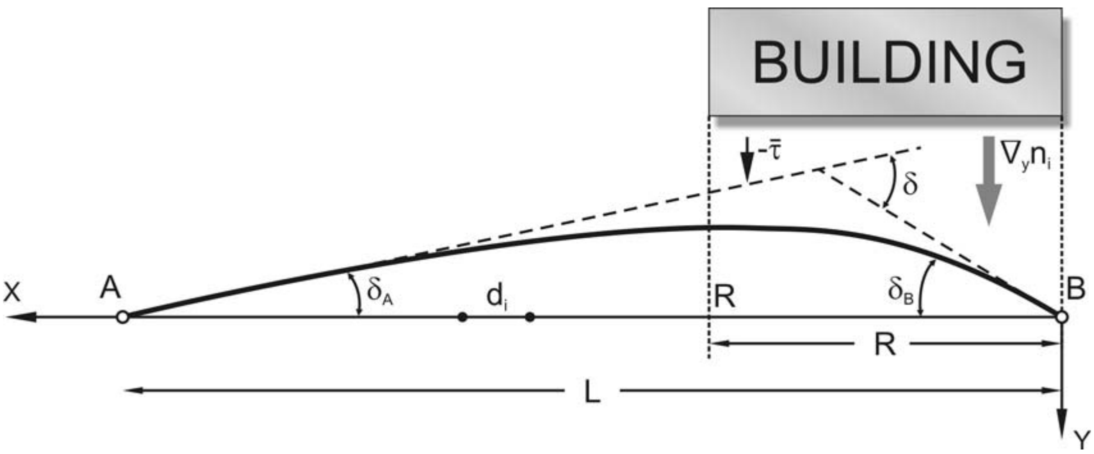

Fig. 2. Influence of horizontal refraction at the end sector of aiming line R.

$$\delta_A = -\frac{1}{L}\int_0^L \frac{1}{n}\frac{\partial n}{\partial y} x \cdot dx = -\frac{1}{2}\frac{R}{L}\sum_0^R d_i \nabla_y n_i = -\frac{1}{2}\frac{R^2}{L}\nabla_y^{sr} n \quad \textbf{(3)}$$

$$\delta_B = \frac{\left(L - \frac{R}{2}\right)}{L}\int_L^0 \frac{1}{n}\frac{\partial n}{\partial y} \cdot dx = \left(1 - \frac{R}{2L}\right)\sum_R^B d_i \nabla_y n_i = \left(1 - \frac{R}{2L}\right) R \cdot \nabla_y^{sr} n \quad \textbf{(4)}$$

4. INFLUENCE OF REFRACTION FOR FREELY LOCATED TEMPERATURE FIELD

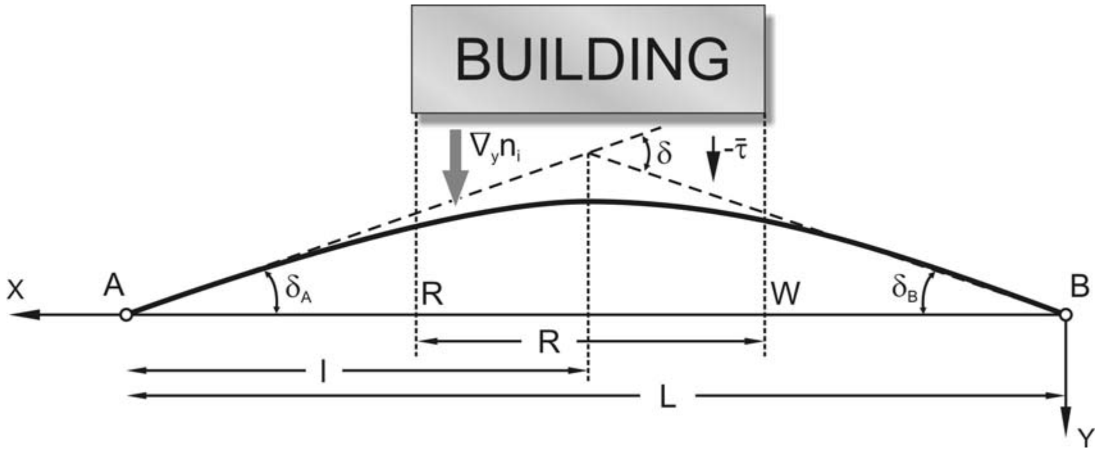

Fig. 3. Influence of refraction of a freely located sector R of aiming line radius course.

$$\delta_A = \frac{L-1}{L}\int_w^R \frac{1}{n}\frac{\partial n}{\partial y} dx = \frac{L-1}{L}\sum_w^R d_i \nabla_y n_i = -\frac{L-1}{L} R \cdot \nabla_y^{sr} n \quad \textbf{(5)}$$

$$\delta_B = \frac{1}{L}\int_w^R \frac{1}{n}\frac{\partial n}{\partial y} dx = \frac{1}{L}\sum_w^R d_i \nabla_y n_i = \frac{1}{L} R \cdot \nabla_y^{sr} n \quad \textbf{(6)}$$

5. TOTAL INFLUENCE OF REFRACTION AT BEGINNING AND END SECTOR OF AIMING LINE

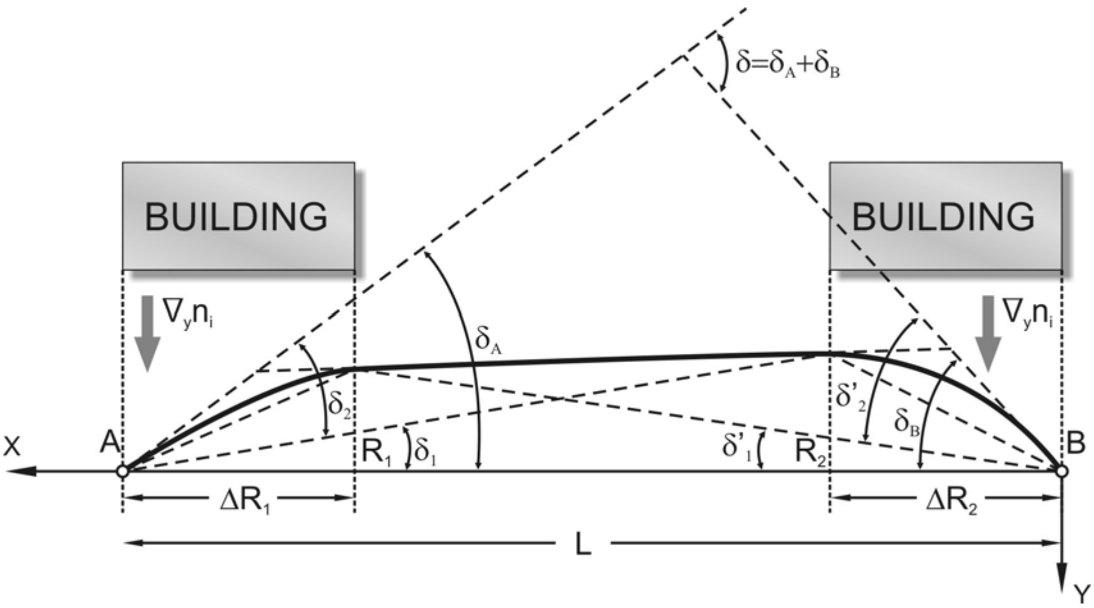

Fig. 4. Total influence of refraction at beginning- ΔR_1 and end - ΔR_2 sector of aiming line.

$$\delta_A = \delta_1 + \delta_2 = -\left[\frac{L - \left(L - \frac{\Delta R_2}{2} \right)}{L} \int_B^{R_2} \frac{1}{n} \frac{\partial n}{\partial y} dx + \frac{\left(L - \frac{\Delta R_1}{2} \right)}{L} \int_{R_1}^{A} \frac{1}{n} \frac{\partial n}{\partial y} dx \right] =$$

$$= -\left[\frac{\Delta R_2}{2L} \sum_B^{R_2} d_i \nabla_y n_i + \frac{2L - \Delta R_1}{2L} \sum_{R_1}^{A} d_i \nabla_y n_i \right] = \quad \textbf{(7)}$$

$$= -\left[\frac{\Delta R_2^2}{2L} \nabla_y^{sr} n + \frac{(2L - \Delta R_1)}{2L} \cdot \Delta R_1 \cdot \nabla_y^{sr} n \right]$$

$$\delta_B = \delta_1^{'} + \delta_2^{'} = \frac{L - \left(L - \frac{\Delta R_1}{2} \right)}{L} \int_A^{R_1} \frac{1}{n} \frac{\partial n}{\partial y} dx + \frac{\left(L - \frac{\Delta R_2}{2} \right)}{L} \int_{R_2}^{B} \frac{1}{n} \frac{\partial n}{\partial y} dx =$$

$$= \frac{\Delta R_1}{2L} \sum_{R_1}^{A} d_i \nabla_y n_i + \frac{\left(L - \frac{\Delta R_2}{2} \right)}{L} \sum_B^{R_2} d_i \nabla_y n_i = \quad \textbf{(8)}$$

$$= \frac{\Delta R_1^2}{2L} \cdot \nabla_y^{sr} n + \frac{\left(L - \frac{\Delta R_2}{2} \right)}{L} \cdot \Delta R_2 \cdot \nabla_y^{sr} n$$

6. Total effect of refraction in local network of industrial plant.

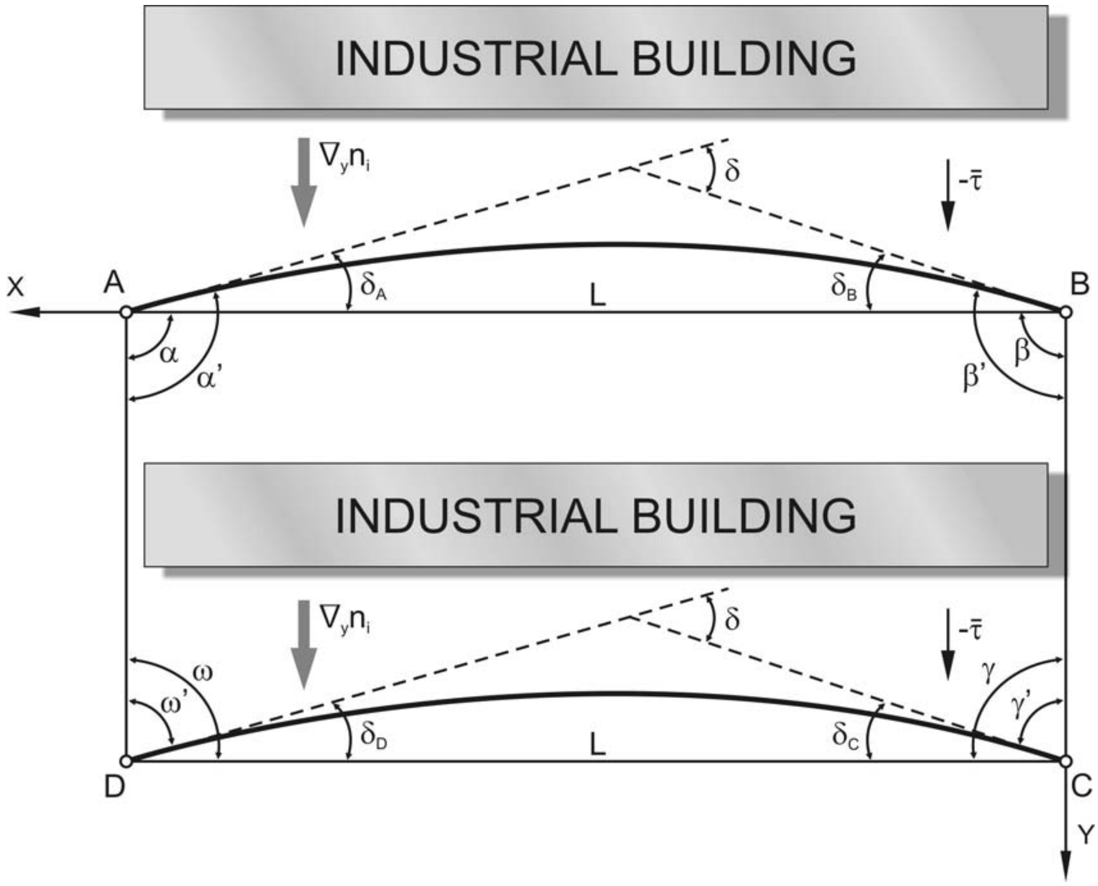

Fig. 5. Influence of refraction on measurement of angles in local network of industrial plant.

$$\sum k^t = \alpha + \delta_A + \beta + \delta_B + \gamma - \delta_C + \varpi - \delta_D = \alpha' + \beta' + \gamma' + \varpi' = \\ = (n-2)200 + \frac{1}{2} L \cdot \nabla_y^{sr} n + \frac{1}{2} L \cdot \nabla_y^{sr} n - \frac{1}{2} L \cdot \nabla_y^{sr} n - \frac{1}{2} L \cdot \nabla_y^{sr} n = \\ = (n-2)200 \qquad \textbf{(9)}$$

$$\alpha = \alpha' - \delta_A \qquad \beta = \beta' - \delta_B \qquad \gamma = \gamma' + \delta_C \qquad \varpi = \varpi' + \delta_D \qquad \textbf{(10)}$$

7. GEOMETRY OF AIMING LINES AND EFFECT OF EDGE REFRACTION

This type of edge refraction (single refraction) can occur in cases when the course of the aiming line runs through two centers of stable but varied temperature – inside a production hall with much higher temperature T_2 and outside of the hall with lower temperature T_1.

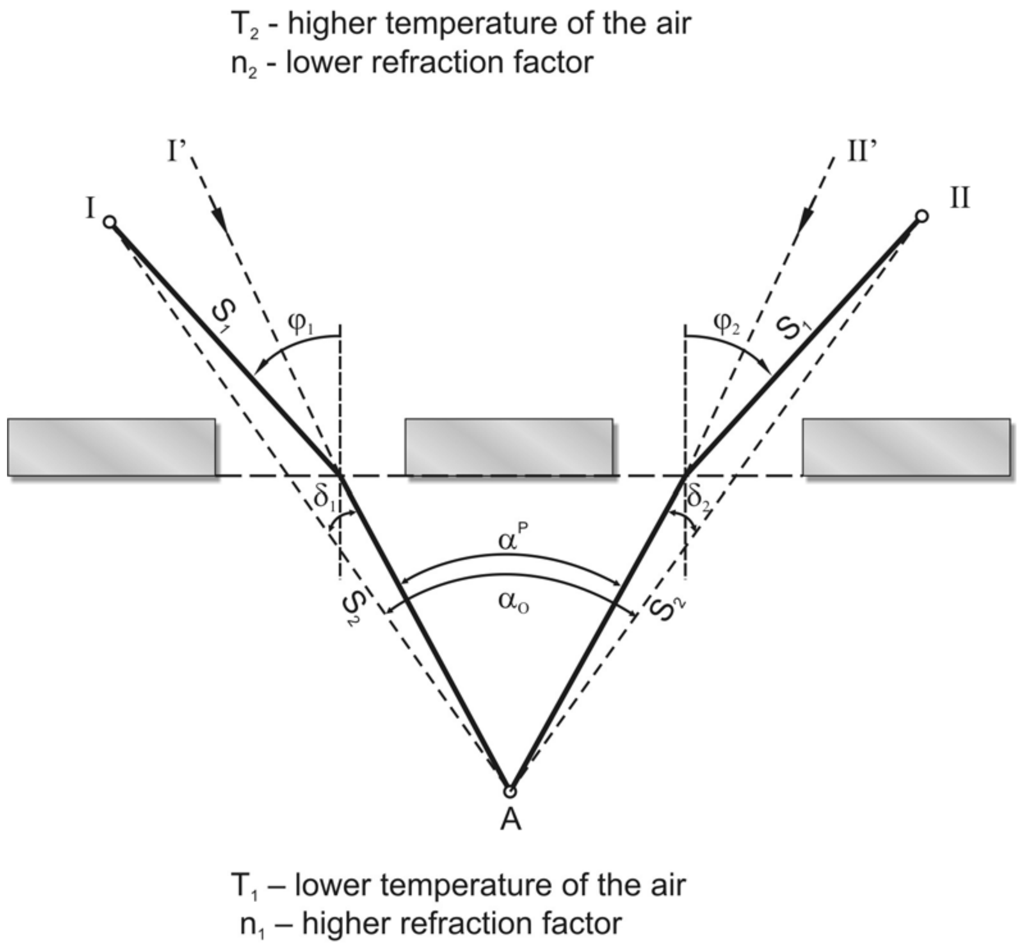

Fig. 6. Influence of edge refraction on the value of the measured horizontal angle.

$$\delta_r = \frac{S_1}{S_2} \cdot 0{,}96 \cdot tg\varphi \cdot \Delta T \cdot 10^{-6} \tag{11}$$

$$\alpha_0 = \alpha^P + \delta_{r1} + \delta_{r2} = \alpha^P + 0{,}96 \cdot \Delta T \cdot 10^{-6} \cdot \left(\frac{S_1}{S_2} tg\varphi_1 + \frac{S_1^{'}}{S_2^{'}} tg\varphi_2 \right) \tag{12}$$

where:

- **S_1,S_2,S_1',S_2'** - **distances**
- **φ** - **angle between perpendicular to the plane of refraction which means:**
- **ΔT** - **difference of temperatures inside and outside the hall in ^{0}C**

8. THE END

Specific conditions of measuring centers of industrial plants especially in time and space of temperature of air and gradients of temperature cause creation of an active refraction field and in consequence the occurrence of refraction of the radius of the aiming line. As concluded from the analyze of formulas, influence of refraction occurrence on results of measurements of horizontal angles is higher when the

instruments are located closer to the sphere of the temperature field. Optic observations will be without an error if the temperature field through which the light beam runs is homogenous and stable , which means when T(x,y,z) = constans.

REFERENCES

Bryś H., 1985. Zur Bestimmung des Einflusses der Horizontalrefraktion bei optischen Alignements. Vermessungstechnik, nr 5/1985.

Bryś H., 1994. Ein moderner Lufttemperaturgradientenmesser mit Digitalanzeige für geodätische Refraktionsforschungen und –bestimmungen. Der Vermessungsingenieur, nr 2/1994.

Moritz H., 1962. Zur Geometrie der Refraktion. Österreichische Zeitschrift für Vermessungswesen, 1(1962).

Ostrovskij A. Ł., 1992. Bokovaja refrakcja i stvornije nabludenija pri inzynerno-geodezicceskich robotach. Zeszyty Naukowe Politechniki Rzeszowskiej, Zeszyt 17, Budownictwo i Inżynieria Środowiska.