Thermospheric density estimation using SLR observations to very low Earth orbiters

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Introduction

**INSIGHT project:** Interactions of Low-orbiting Satellites with the Surrounding Ionosphere and Thermosphere

IAPG-TUM München; DGFI-TUM München; GFZ Potsdam; IfE, University Hannover

**Main objectives of the project:**
- investigate the thermosphere dynamics at low orbits,
- address the ionosphere – thermosphere coupling,
- study the interactions of thermosphere and ionosphere with sensor systems of geodetic space missions.
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DGFI-TUM contributions to INSIGHT:

- Sensitivity analysis of SLR observations to thermospheric models
- Estimation of thermospheric parameters from SLR observations
- Combined estimation including non-spherical satellites such as GOCE and SWARM
- Assimilation of electron density measurements into a thermosphere-ionosphere coupling model
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Available/processed SLR observations

- LEO satellites are hard to observe (fast rotation of telescopes necessary)
- Spherical satellites have very short mission duration (few months to 1-2 years)
Available/processed SLR observations

Processed period for ANDE-Pollux: 49 days (3.5-day arcs)

Atmospheric Neutral Density Experiment (ANDE)

Characteristics of ANDE-Pollux (P):
- Spherical
- Altitude ≈ 350 km
- Inclination ≈ 51.6 degree
- Mass ≈ 27.442 kg
- Center of mass offset ≈ 223.98 mm
- Mission duration (lifetime) from July 2009 to March 2010

The ANDE spheres Castor (left) and Pollux (right)
Equation of motion of LEO satellites

\[ \ddot{r}_{\text{sat}}(t) = a_{\text{KEP}}(t) + a_{\text{GE}}(t) + a_{\text{GM}}(t) + a_{\text{GS}}(t) + a_{\text{GT}}(t) + a_{\text{GNT}}(t) + a_{\text{NG}}(t) \]

- \( a_{\text{DG}}(t) \): Direct Gravitational acceleration
- \( a_{\text{KEP}}(t) \): Gravitational acceleration caused by the point-concentrated mass of the Earth (Stokes coefficient \( C_{0,0} \))
- \( a_{\text{GE}}(t) \): Gravitational acceleration caused by the Earth (Stokes coefficients \( C_{n,m}, S_{n,m} \) with \( n, m \in \mathbb{N}^+ \) and \( m < n \))
- \( a_{\text{GM}} \): Gravitational acceleration caused by the Moon
- \( a_{\text{GS}}(t) \): Gravitational acceleration caused by the Sun and other planets
- \( a_{\text{IG}}(t) \): Indirect Gravitational acceleration (indirect effect on the satellite via the Earth)
- \( a_{\text{GT}}(t) \): Gravitational acceleration caused by mass variations due to solid Earth and ocean tides
- \( a_{\text{GNT}}(t) \): Acceleration caused by mass variations due to non-tidal loading effects (e.g., atmospheric, hydrological, oceanic)
- \( a_{\text{NG}}(t) \): Non-Gravitational acceleration
Equation of motion of LEO satellites

\[
\dot{r}_{\text{sat}}(t) = a_{\text{KEP}}(t) + a_{\text{GE}}(t) + a_{\text{GM}}(t) + a_{\text{GS}}(t) + a_{\text{GT}}(t) + a_{\text{GNT}}(t) + a_{\text{NG}}(t)
\]

According to Ciufolini (1987) and Lucchesi (2001; 2002) the non-gravitational acceleration \(a_{\text{NG}}\) can be split into:

- radiation parts (direct solar radiation pressure, Earth albedo, satellite eclipses, Poynting-Robertson effect, Yarkovski-Rubincam effect (anisotropic thermal radiation), Yarkovski-Schach effect (infrared radiation)),
- drag-like parts (atmospheric drag, solar wind, interplanetary dust),
- other parts (e.g., Earth magnetic field, relativistic effect).

For (spherical) LEO satellites the atmospheric drag perturbing acceleration \(a_{\text{D}}\) is the largest one and, thus, the main error source in LEO satellite POD.
Revised aerodynamic perturbation modeling

- ANDE-P → spherical satellite → only drag forces (no side/lift forces)

\[
a_D = -\frac{1}{2} \cdot \frac{A_{eff}}{m} C_D \rho \frac{v_{rel}^2}{m} \hat{u}_D
\]

\[
\hat{u}_D = \frac{v_{rel}}{\|v_{rel}\|}
\]
drag unit vector

\[v_{rel}\]
relative velocity of the satellite w.r.t. the thermosphere

\[C_D\]
thermospheric drag coefficient, describing the interaction of the atmosphere with the satellite surface

\[\rho\]
integrated thermospheric (neutral) density

\[A_{eff}\]
effective satellite cross-section area interacting with the atmosphere

\[m\]
satellite mass
Refined aerodynamic perturbation modeling

- ANDE-P → spherical satellite → only drag forces (no side/lift forces)

\[ \mathbf{a}_D = -\frac{1}{2} \cdot f_s \cdot \frac{A_{eff}}{m} C_D \rho v_{rel}^2 \mathbf{\hat{u}}_D \]

\[ \mathbf{\hat{u}}_D = \frac{v_{rel}}{\|v_{rel}\|} \] drag unit vector

\[ v_{rel} \] relative velocity of the satellite w.r.t. the thermosphere

\[ C_D \] thermospheric drag coefficient, describing the interaction of the atmosphere with the satellite surface

\[ \rho \] integrated thermospheric (neutral) density

\[ A_{eff} \] effective satellite cross-section area interacting with the atmosphere

\[ m \] satellite mass

\[ f_s \] Lagrange scaling factor → this parameter is estimated!

How does the estimated scaling parameter be interpreted?
Refined aerodynamic perturbation modeling

- ANDE-P → spherical satellite → only drag forces (no side/lift forces)

\[ a_D = -\frac{1}{2} \cdot f_s \cdot \frac{A_{eff}}{m} C_D \rho v_{rel}^2 \hat{u}_D \]

To which of the terms in the equation above the scaling factor can be associated?

- The area-to-mass relation \( A_{eff}/m \) is for spherical satellites usually the most accurate parameter.
- The uncertainties in the thermospheric density and the drag coefficient \( C_D \) are significant under all conditions of solar and geomagnetic activity and at all latitudes and thermospheric altitudes.
- At altitudes below 350 km it is assumed that the thermospheric density is the least accurate parameter.
- Thus, we assume the scaling parameter \( f_s \) is associated with \( \rho \).
Computation of the thermospheric density $\rho$

- Implemented thermospheric models

<table>
<thead>
<tr>
<th>Thermospheric model</th>
<th>Reference</th>
<th>thermospheric constituents (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTM2013</td>
<td>(Bruinsma et al., 2008)</td>
<td>H, He, O, N2, O2, Ar</td>
</tr>
<tr>
<td>JB2008</td>
<td>(Bowman et al., 2008)</td>
<td>H, He, O, N2, O2, Ar</td>
</tr>
<tr>
<td>CIRA86</td>
<td>(Hedin et al., 1988)</td>
<td>H, He, N, O, N2, O2, Ar</td>
</tr>
<tr>
<td>NRLMSISE00</td>
<td>(Picone et al., 2002)</td>
<td>H, He, N, O, N2, O2, Ar, anomalous oxygen</td>
</tr>
</tbody>
</table>

- Implemented horizontal wind model: HWM14 (Drob et al., 2015)

"hot" atomic oxygen and ionospheric atomic oxygen ions O+ (to be considered at altitudes > 500 km)
Computation of the drag coefficient $C_D$

Assumption 1  →  Free molecular flow: no inter-molecular collisions at satellite altitudes > 150 km (based on the Knudsen number)

Assumption 2  →  Sentman’s GSI model: interaction between gas and satellite surface at altitudes < 500 km

Assumption 3  →  Satellite surface is covered with a layer of absorbed atomic oxygen → fully diffuse reflection of the gas particles with full accommodation ($\alpha = 1$; explanation on next slide)

Assumption 4  →  Thermal flow: incident flow at satellite surface is superposition of random thermal molecule velocity (Maxwell-Boltzmann distribution) and bulk velocity ($v_{rel}$)

Assumption 5  →  Maxwell-Boltzmann velocity of re-emitted particles

Assumption 6  →  $T_w = 300 \, K$
Computation of physically-based drag coefficient $C_D$

- Based on the assumptions, $C_D$ can be computed according to

$$C_D^{(sp)} = \frac{4s_k^4 + 4s_k^2 - 1}{2s_k^4} \text{erf}(s_k) + \frac{2s_k^2 + 1}{\sqrt{\pi}s_k^3} e^{-s_k^2} + \frac{2\sqrt{\pi}}{34s_k} \sqrt{\frac{T_W}{T_\infty}}$$

Key parameter 1 $\rightarrow$ Molecular speed ratio $s_k = \frac{v_{rel}}{v_{m,k}}$ with $v_{m,k} = \sqrt{\frac{2RT_\infty}{m_k}}$ (most probable molecular velocity for $k$'th constituent with atomic mass $m_k$ (using gas kinetic theory; Maxwell-Boltzmann)

Key parameter 2 $\rightarrow$ Satellite relative velocity w.r.t. thermosphere

$$v_{rel} = v_{sat} - \omega_e \times x_{sat} - v_{wind}$$

Key parameter 3 $\rightarrow$ Energy accommodation coefficient $\alpha = \frac{E_i - E_r}{E_i - E_w}$ which quantifies the amount of energy exchange between gas and surface

Key parameter 4 $\rightarrow$
Computation of physically-based drag coefficient $C_D$ (3/3)

- Based on the assumptions, $C_D$ can be computed according to

$$C_{D,k}^{(sp)} = \frac{4s_k^4 + 4s_k^2 - 1}{2s_k^4} \text{erf}(s_k) + \frac{2s_k^2 + 1}{\sqrt{\pi}s_k^3} e^{-s_k^2} + \frac{2\sqrt{\pi}}{34s_k} \frac{T_w}{T_\infty}$$

Key parameter 1

Key parameter 2

Key parameter 3

Key parameter 4

Key parameter 1 $\rightarrow$ ADSORPTION

Key parameter 2 $\rightarrow$ SPECULAR REFLECTION

Key parameter 3 $\rightarrow$ DIFFUSE REFLECTION

Key parameter 4 $\rightarrow$ Satellite surface and thermospheric temperatures $T_w$ and $T_\infty$

Energy accommodation coefficient $\alpha = \frac{E_i - E_r}{E_i - E_w}$ which quantifies the amount of energy exchange between gas and surface.

$E_i$, $E_r$, $E_w \rightarrow$ kinetic energies of incident, reflected and re-emitted molecules with temperature of the surface (wall)
ANDE-Pollux observations

- Geographic distribution
ANDE-Pollux observations

- Temporal distribution: 49 days from end of August to beginning of October 2009
Physically modeled $C_D$ for ANDE-Pollux

From Nicholas et al. (2007), we get $C_D = 2.1123 \pm 0.00763$ based on different assumptions and different thermospheric model (thermospheric temperature); comparable with the average value from above.
ANDE-Pollux solution setup

- General dynamic models: IERS Conventions 2010

<table>
<thead>
<tr>
<th>Estimated parameters</th>
<th>Temporal resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keplerian elements</td>
<td>One set per arc (initial epoch)</td>
</tr>
<tr>
<td>Solar radiation pressure coefficient</td>
<td>One per arc</td>
</tr>
<tr>
<td>Albedo coefficient</td>
<td>One per arc</td>
</tr>
<tr>
<td>Empirical coefficients (CPRs)</td>
<td>One set per arc (sin/cos; along-/cross-track)</td>
</tr>
<tr>
<td>Lagrange scaling coefficients</td>
<td>Four per day (6-hour resolution; along-tr.)</td>
</tr>
</tbody>
</table>
Integrated sensitivity on thermospheric models

- Impact of thermospheric models on SLR orbit computation (RMS of observation residuals)

- Mostly low impact on orbit variance factors since estimated scaling coefficients compensate differences in thermospheric models

- Variance factors hard to compare since outlier detection is done separately for each solution setup
Estimated Lagrange scaling coefficients $f_s$

- If the scaling coefficient is equal to 1 no SLR observations were available
- NRLMSISE00 and CIRA86 agree very well; more recent models show offset (especially at the end of the time series)
Modeled thermospheric densities $\rho$

- JB2008 shows in general the lowest density distribution values
- Large offset of JB2008 w.r.t. CIRA86 and NRLMSISE00; smaller offset w.r.t. DTM2013
Scaled thermospheric densities $f_s \cdot \rho$

- In general, CIRA86 and NRLMSISE00 are scaled to more recent models.
- No offset change for JB2008 and DTM2013.
- Density variations of JB2008 and DTM2013 are reduced.
Summary

- Refined aerodynamic perturbation modeling in DGFI-TUM’s POD software package allows us to estimate integrated absolute thermospheric densities by considering:
  - GSI-models of Schamberg and Sentman
  - thermospheric models CIRA86, NRLMSISE00, JB2008 and DTM2013
  - horizontal wind model HWM2014
  - estimation of Lagrange scaling factors in any temporal resolution (currently 6h)
- Setup of the software package for the processing of 49 days of ANDE-Pollux SLR observations to perform a sensitivity analysis to the thermospheric density variations
- SLR observations are sensitive to different thermospheric densities
- CIRA86 and NRLMSISE00 are scaled towards the more recent models JB2008 and DTM2013
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Thanks for your attention