Stochastic assessment of GPS measurements for instantaneous carrier phase positioning

1. MOTIVATION

For optimal estimation of the unknown parameters in the positioning model both **functional** and **stochastic** models need to be carefully defined. Whilst the GNSS functional model was the subject of detailed research conducted over the past twenty years and it is well documented for different types of positioning applications, the issue of proper definition of stochastic model have been undertaken in recent few years and is still an open research problem. It is a common approach to assuming a constant accuracy of GNSS measurements and neglecting cross and time correlation between them. This is reflected in the design of variance-covariance matrix which is usually diagonal matrix with a priori defined entries. Especially with respect to instantaneous applications which are characterized by weakening model strength, unrealistic or simplified definition of stochastic properties of observations causes that the performance of ambiguity resolution and position estimation can be limited.

In this study we investigate the methods of stochastic assessment of GNSS observations and its impact to derive precise positioning model. Especially the stochastic modelling procedure based on observations from GNSS **signal generator** has been developed to determine individual components of the variance-covariance matrix. The results of **ambiguity resolution** as well as the **positioning accuracy** shows that the utilizing the individual empirical stochastic model of observations increase the reliability of **instantaneous performance**.

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2. EXPERIMENT DESIGNE

Stochastic properties of observations were determined for GPS receiver **Leica GX1230GG** (S. No. 468950) based on:
GPS Signal generator observations: Spirent GNSS Generator
Zero-baseline observations: WUT1 stations located on the roof of Warsaw University of Technology Main Building; reference receiver - Leica GX1230GG (S. No. 466566)

The data used:
Observations: GPS - L1 / L2 / C1 / P2
Time: 1.01.2016 / 24^h / sample rates: 30 sec. (2880 epoch) / cut-off angle: **1 deg.**

Compare models:
#1 Standard model: $\sigma_{L1/2} = 0.003$ m; $\sigma_{P1/2} = 0.30$ m; $\mathrm{cov}_{P/L} = 0$
#2 Standard elevation model: $\sigma_{Li} \rightarrow$ a= 0.003 m b=0.003 m; $\sigma_{P1/2} = 100 \cdot \sigma_{L1/2}$; $\mathrm{cov}_{P/L} = 0$
#3 Combined elevation model: $\sigma_{Li} / \sigma_{Pi} \rightarrow$ a,b - individual determined models; $\mathrm{cov}_{P/L} = 0$
#4 Combined elevation + cross-correlation model: $\sigma_{Li} / \sigma_{Pi} / \rightarrow$ a,b - individual determined models; $\mathrm{cov}_{L1\,L2}$ - individual determined models

Positioning model:
Observations: zero-baseline
Functional model: Geometry-Based Double-Differenced
Stochastic model: Ionosphere-Fixed Troposphere Fixed
Ambiguity resolution: **Instantaneous** (one-epoch resolution)
ILS estimation method: MLAMBDA
ILS validation test: R-ratio

3. METHODOLOGY

GPS receiver testing procedure consists of two tests:
1. **Signal generator test** - the same 24-hours observations were generated two times; based on that double-differenced code and carrier-phase residuals for zero-baseline were calculated; the residuals were used to determine **constant part** of measurement noise and **cross-correlations** of GPS observations;
2. **Zero-baseline test** - the same type of GPS receivers were connected to the one antenna; based on that the 24-hours of double-differenced code and carrier-phase residuals for zero-baseline were calculated; the residuals were used to determine **elevation-dependent** part of measurement noise;

Code (P) and carrier-phase (L) pseudorange model:

$$P = \|\mathbf{R}\| + \delta I + \delta T + \delta m + \delta\rho_{sat} + c(\delta t_{rec} - \delta t_{sat}) + \varepsilon_P$$
$$L = \|\mathbf{R}\| - \delta I + \delta T + \delta m + \delta\rho_{sat} + c(\delta t_{rec} - \delta t_{sat}) + \lambda N + \varepsilon_L$$

Double-differenced code and carrier-phase observation residuals for a zero-baseline:

$$\nabla\Delta P - \|\nabla\Delta\mathbf{R}\| = \nabla\Delta\varepsilon_P \qquad \nabla\Delta L - \|\nabla\Delta\mathbf{R}\| - \lambda N = \nabla\Delta\varepsilon_L$$

Variance and **covariance** of code and carrier-phase noise have been determined according to formulas:

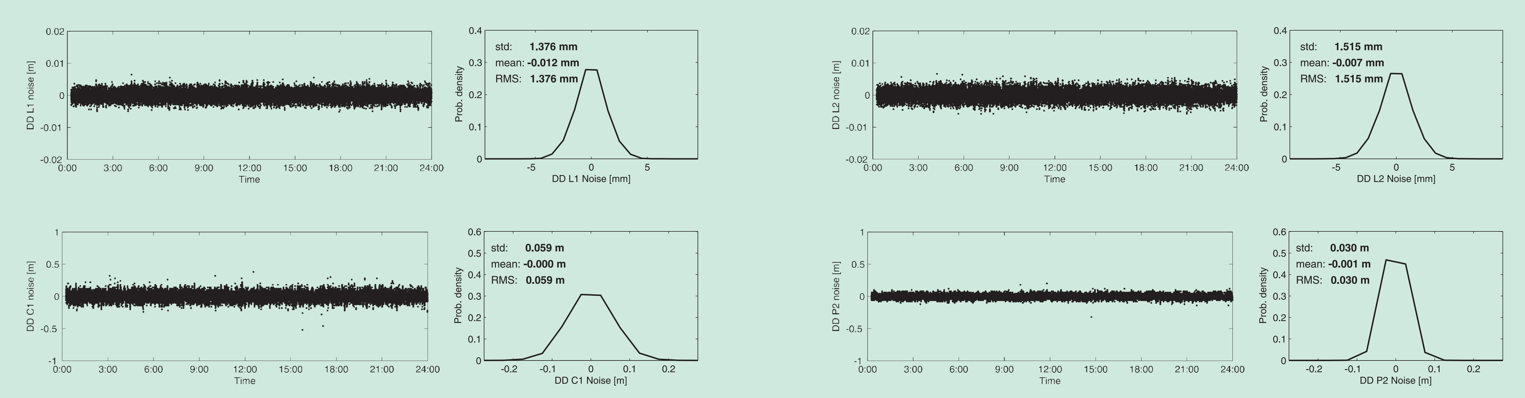
$$\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2 \qquad \mathrm{Cov}(X,Y) = \mathrm{E}(X \cdot Y) - \mathrm{E}(X)\mathrm{E}(Y) \qquad \mathrm{Var}(P) = \mathrm{Var}(\nabla\Delta P)/4$$

Observation noise model - **elevation dependency**:

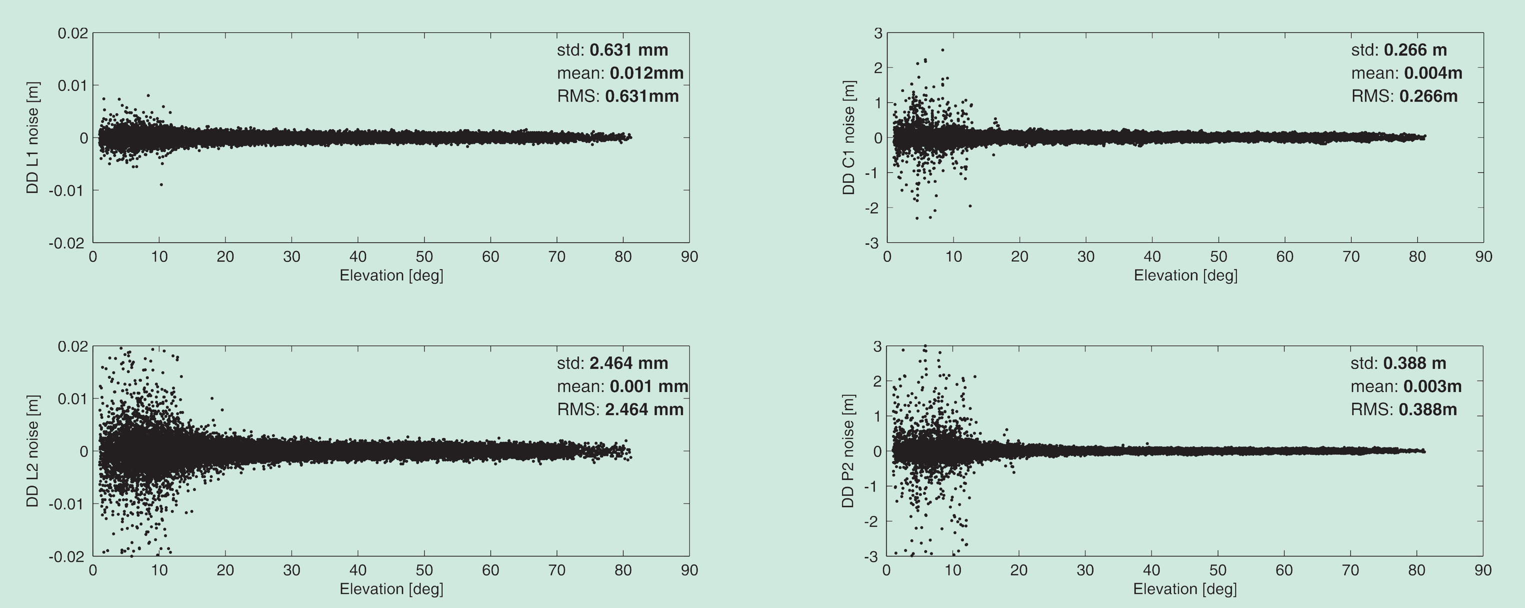
$$\mathrm{Var}(X) = (a + b/\sin el)^2$$

4. TEST RESULTS

Signal generator test:

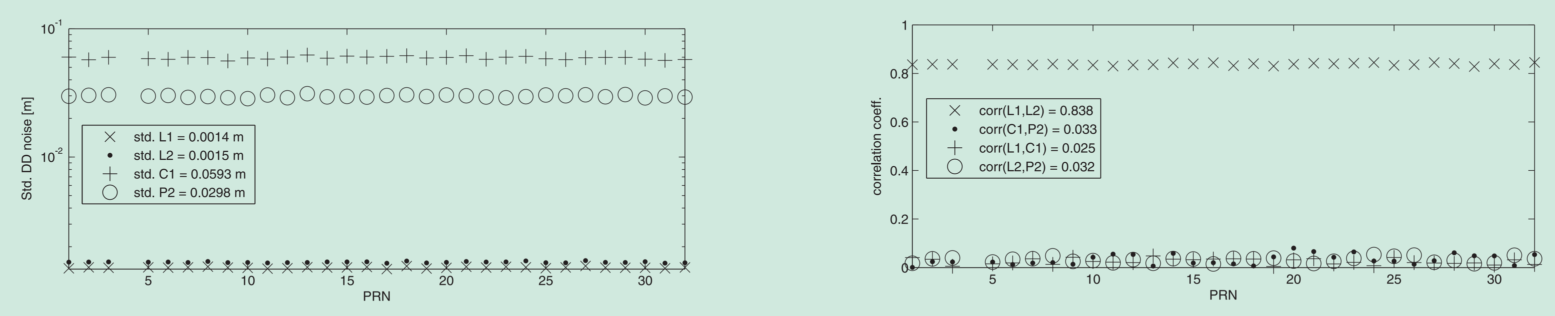


Zero-baseline test:

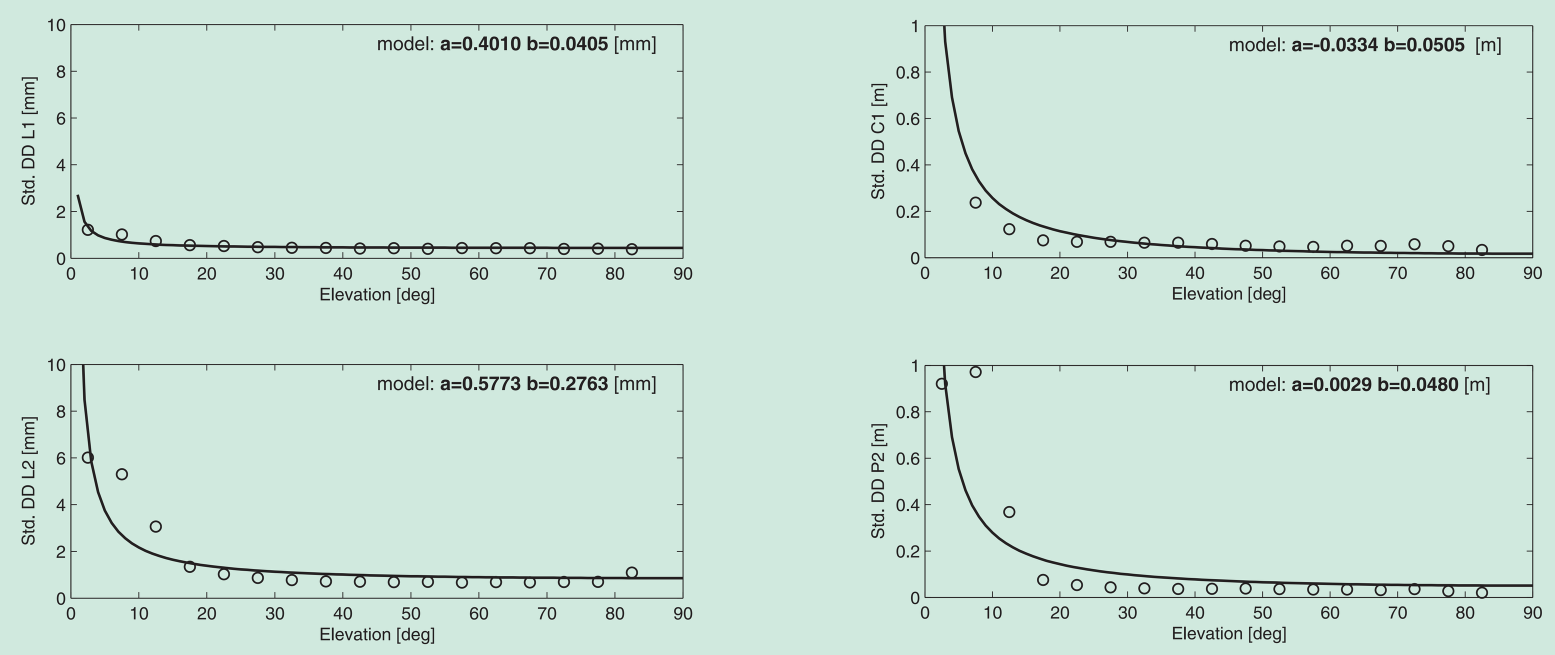


5. STOCHASTIC PROPERTIES

Signal generator test:



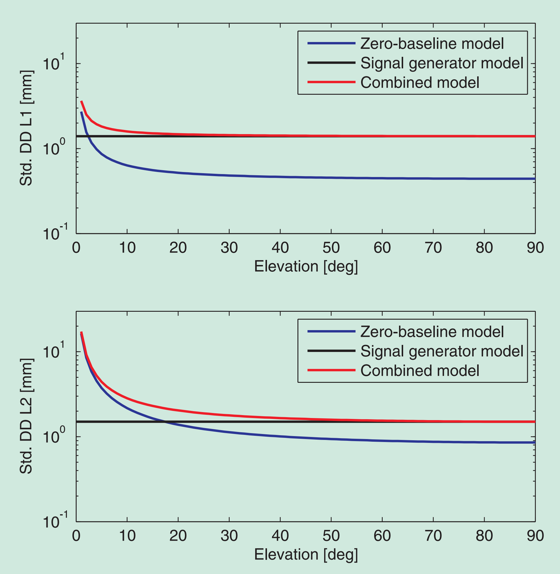
Zero-baseline test:



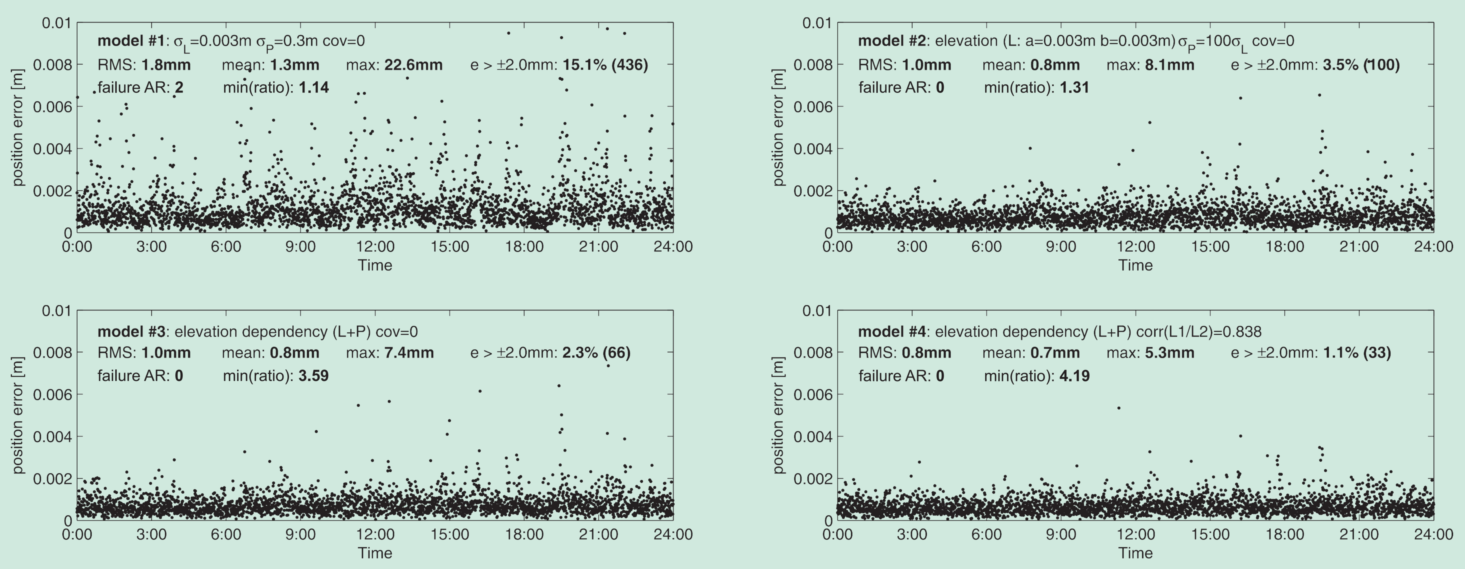
6. STOCHASTIC MODEL & POSITIONING PERFORMANCE

Stochastic model:

Individual combined model =
elevation dependency + constant part



Positioning performance:



7. CONCLUSIONS

- The stochastic modelling of individual components of the variance-covariance matrix of observation noise allows for **increased reliability** of solution in both the **ambiguity resolution** and **solution accuracy** aspects;
- The use of individual determined models of observations noise and cross-correlation is especially important for kinematic application based on a single observational epoch where **instantaneous** stochastic properties of observations could be significantly differ from parameters of standard stochastic model;
- **The individual combined model** (elevation dependency + constant part) based on two test: signal generator test and zero-baseline test shows good capability to model the variance as well as the cross-correlation;
- Presented approach of stochastic modelling of GNSS observation can be important part in a comprehensive **calibration procedure** of GNSS equipment.

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