ON SELECTED ISSUES OF COMBINED MULTI GNSS (GPS, BDS, GALILEO) PRECISE POSITIONING

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Motivation:

• Integration of multi GNSS observations both in precise relative and precise point positioning is nowadays a subject of extensive studies.
• Potential improvement concerns performance in obstructed environment – limited satellite visibility in, e.g., urban canyons
• Still the open question is how to optimally combine multi GNSS signals in a single functional model.

Goal:

• Select the optimal method of multi GNSS observations combining in the RTK positioning on the basis of GPS+Galileo observations.
• Assess the application of BDS system to support medium range multi-baseline BDS+GPS RTK positioning.
Multi-GNSS precise relative positioning may be performed with:

1. **loosely combined (LC) approach**
   a) functional model utilizes separate pivot (reference) satellites for each GNSS system,
   b) decreases the number of double-differenced observables,
   c) no ISB.

2. **tightly combined (TC) approach**
   a) functional model utilizes a single pivot (reference) satellite for all GNSS systems
   b) mixed double-differenced observables,
   c) ISB has to be estimated or provided from earlier calibration.

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Multi-GNSS loose combined ionosphere and troposphere weighted functional model

\[
\begin{align*}
\lambda_{f_1} \varphi_{kl,f_1}^{mn} - \rho_{kl}^{mn} - (\alpha_k^m ZTD_k - \alpha_k^n ZTD_k - \alpha_l^m ZTD_l + \alpha_l^n ZTD_l) + I_{kl}^{mn} - \lambda_{f_1} N_{kl,f_1}^{mn} &= 0 \\
P_{kl,f_G}^{mn} - \rho_{kl}^{mn} - (\alpha_k^m ZTD_k - \alpha_k^n ZTD_k - \alpha_l^m ZTD_l + \alpha_l^n ZTD_l) - I_{kl}^{mn} &= 0 \\
\lambda_{f_2} \varphi_{kl,f_2}^{mn} - \rho_{kl}^{mn} - (\alpha_k^m ZTD_k - \alpha_k^n ZTD_k - \alpha_l^m ZTD_l + \alpha_l^n ZTD_l) + I_{kl}^{mn} (f_1^2 / f_2^2) - \lambda_{f_2} N_{kl,f_2}^{mn} &= 0 \\
P_{kl,f_G}^{mn} - \rho_{kl}^{mn} - (\alpha_k^m ZTD_k - \alpha_k^n ZTD_k - \alpha_l^m ZTD_l + \alpha_l^n ZTD_l) - I_{kl}^{mn} (f_1^2 / f_2^2) &= 0
\end{align*}
\]

where \( \lambda \) is the signal wavelength, \( \varphi \) is the carrier phase observable, \( P \) is the pseudorange and \( f \) denotes the frequency, subscripts \( k, l \) and superscripts \( m, n \) denote stations and satellites, respectively, \( \rho \) is the geometric range, \( N \) stands for integer DD ambiguity, \( I \) is the DD ionospheric delay, \( \alpha \) is the troposphere mapping function coefficient and \( ZTD \) is the residual zenith tropospheric delay at each station.
Tight integration (combination) requires taking into account differences in:

- receiver hardware delays - *inter-system bias (ISB)*

\[
\lambda\varphi_{kl}^{ij} = \rho_{kl}^{ij} + T_{kl}^{ij} - I_{kl}^{ij} + \lambda N_{kl}^{ij} + \delta_{kl}^{(G-E)} + \epsilon_{kl,\varphi}^{ij}
\]

integer part

\[
P_{kl}^{ij} = \rho_{kl}^{ij} + T_{kl}^{ij} + I_{kl}^{ij} + d_{kl}^{(G-E)} + \epsilon_{kl,P}^{ij} = 0
\]

fractional part

\[
\delta_{kl}^{(G-E)} = \tilde{\delta}_{kl}^{(G-E)} + \lambda M_{kl}^{GE}
\]

We can recognize two general strategies for handling the ISB in the tightly combined (TC) multi-GNSS positioning:

A. phase and code receiver inter system biases can be treated as additional parameters in the relative positioning model estimation - unknown ISB (TC-U);

B. phase and code GNSS observations can be corrected with previously calibrated receiver ISB values - ISB calibrated/fixed (TC-C).

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1. GPS-Galileo ISB studies

Phase and code receiver Inter System Biases for non-homogenous pair of receivers cannot be neglected.

Fig. Phase and code ISB time series obtained using all available GPS and Galileo satellites for different receiver pairs.
Multi-GNSS tightly combined RTK ionosphere and troposphere weighted model with calibrated ISB

\[
\begin{align*}
\lambda_1 \tilde{\phi}_{kl,f_1}^{m_G i_E} - \rho_{kl}^{m_G i_E} - (\alpha_k^{m_G} ZTD_k - \alpha_k^{i_E} ZTD_k - \alpha_l^{m_G} ZTD_l + \alpha_l^{i_E} ZTD_l) + I_{kl}^{m_G i_E} - \lambda_1 \overline{N}_{kl,f_1}^{m_G i_E} &= 0 \\
\tilde{P}_{kl,f_1}^{m_G i_E} - \rho_{kl}^{m_G i_E} - (\alpha_k^{m_G} ZTD_k - \alpha_k^{i_E} ZTD_k - \alpha_l^{m_G} ZTD_l + \alpha_l^{i_E} ZTD_l) - I_{kl}^{m_G i_E} &= 0 \\
\lambda_2 \tilde{\phi}_{kl,f_2}^{m_G i_E} - \rho_{kl}^{m_G i_E} - (\alpha_k^{m_G} ZTD_k - \alpha_k^{i_E} ZTD_k - \alpha_l^{m_G} ZTD_l + \alpha_l^{i_E} ZTD_l) + I_{kl}^{m_G i_E}(f_1^2/f_2^2) - \lambda_2 \overline{N}_{kl,f_2}^{m_G i_E} &= 0 \\
\tilde{P}_{kl,f_2}^{m_G i_E} - \rho_{kl}^{m_G i_E} - (\alpha_k^{m_G} ZTD_k - \alpha_k^{i_E} ZTD_k - \alpha_l^{m_G} ZTD_l + \alpha_l^{i_E} ZTD_l) - I_{kl}^{m_G i_E}(f_1^2/f_2^2) &= 0
\end{align*}
\]

where \(\lambda\) is the signal wavelength, \(\phi\) is the carrier phase observable, \(P\) is the pseudorange and \(f\) denotes the frequency, subscripts \(k, l\) and superscripts \(m, n\) denote stations and satellites, respectively, \(\rho\) is the geometric range, \(N\) stands for integer DD ambiguity, \(I\) is the DD ionospheric delay, \(\alpha\) is the troposphere mapping function coefficient and \(ZTD\) is the residual zenith tropospheric delay at each station,

- DD ambiguities are combined with unknown integer part of the phase ISB and estimated as a single parameter for each frequency \((\overline{N}_{kl,f_1}^{m_G i_E}, \overline{N}_{kl,f_2}^{m_G i_E})\).
- ISB affects multi-GNSS relative positioning and cannot be neglected.
GINPOS - GNSS processing scientific software

Processing modes:
- Single Point Positioning/DGPS
- Static/kinematic relative positioning
- **Instantaneous RTK positioning** (ionosphere and troposphere weighted model)
- PPP

Supported GNSS systems:
- Galileo
- GPS
- BDS
- EGNOS/WAAS

Adjustment model:
- Sequential Least Squares Adjustment with a priori parameter constraining and elimination
Experiment #1: **Instantaneous** Galileo+GPS RTK positioning

In order to examine different approaches for multi GNSS data combining, several processing strategies were established:

1. GPS L1 & L5 (**GPS**)
2. Tightly combined L1/E1 & L5/E5a GPS+Galileo with unknown (estimated) ISBs (**TC-U**)
3. Tightly combined L1/E1 & L5/E5a GPS+Galileo with previously calibrated (fixed) ISBs (**TC-C**)
4. Loosely combined L1/E1 & L5/E5a GPS+Galileo (**LC**)

- The experiment is based on instantaneous RTK performed in the post-processing mode using multi-system (Galileo and GPS) dual-frequency observations.
- In this study we utilize full constellations of Galileo and modernized GPS with L5 signals.
- The observational data were obtained from Spirent GSS7700/7800 hardware GNSS signal simulator at the ESTEC/ESA center.
Experiment #1: **Instantaneous Galileo+GPS RTK positioning**

Fig. Redundancy (dotted lines) and the number of parameters (solid lines) in the ionosphere-weighted multi-GNSS multi-baseline model.

Fig. Number of satellites during session.
Experiment #1: **Instantaneous Galileo+GPS RTK positioning**

Fig. Rover coordinate residuals for ambiguity float (grey) and fixed (green) solutions

Table: Empirical standard deviations of the rover coordinates obtained in ambiguity float/fixed solutions.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>single baseline solution</th>
<th>multi-baseline solution</th>
<th>ASR [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N  [cm]</td>
<td>E  [cm]</td>
<td>U  [cm]</td>
</tr>
<tr>
<td>GPS (#1)</td>
<td>41.6/0.5</td>
<td>22.3/0.3</td>
<td>65.8/1.6</td>
</tr>
<tr>
<td>Tightly combined GPS+Galileo TC-U (#2)</td>
<td>26.7/0.9</td>
<td>29.5/1.1</td>
<td>47.9/5.3</td>
</tr>
<tr>
<td>Tightly combined GPS+Galileo TC-C (#3)</td>
<td><strong>24.4/0.8</strong></td>
<td><strong>15.8/0.4</strong></td>
<td><strong>36.8/1.8</strong></td>
</tr>
<tr>
<td>GPS+Galileo (#4) LC</td>
<td>26.0/0.5</td>
<td>16.8/0.3</td>
<td>38.4/1.3</td>
</tr>
</tbody>
</table>

Fig. Ambiguity success rates for the defined strategies
Experiment #2: **Instantaneous BDS+GPS RTK positioning**

In order to examine presented approaches for multi GNSS data combining, three processing strategies were established. First and second strategies utilize application of single GNSS system (GPS or BDS). Last strategy utilizes loose integration approach of GPS and BDS signals in precise relative positioning:

1. GPS (L1 & L2)
2. BDS (B1 & B2)
3. Combined GPS + BDS (L1 & L2 + B1 & B2)

Observational data were provided by Dr Maorong Ge and Dr Xingxing Li from GFZ which is acknowledged.
**Experiment #2: Instantaneous BDS+GPS RTK positioning**

- High variations in the number of satellites can be seen when single system (GPS or BDS) was applied.
- We can expect some problems with the solutions at the beginning of the session, especially when only BDS signals were applied. At that time the highest PDOP values were observed.

![5° elev. mask](image)

![25° elev. mask](image)

**Fig. 2** Number of satellites for applied GNSS systems at 5° elevation mask. [GPS, BDS, GPS+BDS]

**Fig. 4** PDOP values obtained using selected GNSS systems and 5° elevation mask.

**Fig. 3** Number of satellites for applied GNSS systems at 25° elevation mask. [GPS, BDS, GPS+BDS]

**Fig. 5** PDOP values obtained using selected GNSS systems and 25° elevation mask.
In case of high elevation cut-off (>30°) only multi GNSS observations (GPS+BDS) give chance for reliable positioning.

Experiment #2: **Instantaneous BDS+GPS RTK positioning**

Fig. 6 Coordinate residuals obtained in the instantaneous positioning with high elevation cut-off (30 deg.)

Fig. 7 Ambiguity success rate in multi-baseline instantaneous positioning as a function of the elevation cut-off angle.

no solution, ionospheric disturbances
• We assessed several strategies allowing for integration of multi GNSS observations in precise relative positioning.

• In the instantaneous positioning it is recommended to apply lose combining (LC) or introduce previously calibrated ISB in the tight combining (TC-C) approach.

• ISB estimation in the observational model (TC-U) reduces the redundancy of the system and makes validation of the ambiguity resolution very difficult.

• We demonstrated that combined positioning has advantage over single GNSS system solution in the ambiguity resolution domain, especially in harsh observing conditions.

• There are no important differences in ambiguity success rate between GPS and GPS+BDS strategy in relatively good observing conditions (for elevation masks up to 20°). On the other hand, combined GPS+BDS positioning has a clear advantage over single GNSS system in harsh observing conditions (with elevation mask over 20°).

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