Mesoscale mapping functions
(Slant delay mesoscale functions)
We present a prototype of computer module for GPS slant delay determination using data from COAMPS (Coupled Ocean/Atmosphere Mesoscale Prediction System) NRL (Naval Research Laboratory).

COAMPS is a mesoscale non-hydrostatic model of the atmosphere which is run on IA64 Feniks computer cluster in the Department of Civil Engineering and Geodesy of the Military University of Technology.

The slant delay is the result of integrating the ray (eikonal) equation for the spatial function of tropospheric refraction along the GPS wave propagation path. The work is a phase of research concerning the impact of mesoscale atmospheric phenomena on the tropospheric delay of the GPS signal.
Development (some facts)


Slant delay module

Mesoscale model - COAMPS

Subroutine: Refraction

Subroutine: Refraction approximation

Subroutine: Ray path and slant delay integration

Slant delay $\tau$

$$\tau = \int (n-1) ds = 10^{-6} \int N ds$$

$$N = k_1 \frac{p_d}{T} Z_d^{-1} + k_2 \frac{e}{T} Z_w^{-1} + k_3 \frac{e}{T^2} Z_w^{-1}$$

$n$ – refraction (refractive index), $N$ – refractivity, $p_d$ and $e$ partial pressure of dry air and of water vapor, $T$ – temperature, $Z_d$ and $Z_w$ – compressibility of dry air and of water vapor, $k_1$, $k_2$, $k_3$ constants determined experimentally.
Mesoscale model domain

Atmospheric model vertical grids
(Terrain following sigma Z system)

\[ \sigma = z_{top} (z - z_s) / (z_{top} - z_s) \]

- \( z_{top} = H \) – depth of the model domain (31.50 km ~ 10 hPa),
- \( z_s \) – height of topography, \( kka \) – number of levels (\( kka = 30 \)).

Horizontal grids - Lambert Conformal projection.
**Prognostic parameters:**

$\pi$ – Exner pressure (function),

$\theta$ – potential temperature,

$q$ – specific humidity,

$u, v, w$ – velocity components

**Diagnostic parameters:**

$T$ – temperature,

$e$ – partial pressure of water vapor

$p_d$ – partial pressure of dry air
Refractivity approximation

\[ N_{i,j,m}^* = N_{i,j,k-1} + w \cdot (N_{i,j,k} - N_{i,j,k-1}), \quad w = \left( z_m^* - z_{i,j,k-1} \right) / \left( z_{i,j,k} - z_{i,j,k-1} \right) \]

Linear approximation

\[ N_{xyz} = N_{000}(1-x)(1-y)(1-z) + N_{100}x(1-y)(1-z) + N_{010}(1-x)y(1-z) + N_{001}(1-x)(1-y)z + N_{101}x(1-y)z + N_{011}(1-x)yz + n_{110}xy(1-z) + N_{111}xyz \]

\[ x = x / dx, \quad y = y / dy, \quad z = z / dh^* \]

Trilinear interpolation

Polynomial structure of \( N \) makes easy gradient \( \nabla N \) calculation
Ray tracing in the atmosphere. Ray definition.

Propagation of rays are determined by the eikonal equation:

$$\sum_{i=1}^{k} \left( \frac{\partial \varphi(\vec{r})}{\partial r_i} \right)^2 = n^2(\vec{r}), \quad k = 3$$

where the gradient of $\varphi(\vec{r})$ gives the direction of the ray, $n(\vec{r})$ index of atmospheric refraction. The $\varphi(\vec{r})$ function is often called eikonal.
The ray equation is the eikonal equation in a new ray based coordinate

\[ \frac{d}{ds} \left[ n(\vec{r}) \frac{d\vec{r}}{ds} \right] = \nabla n(\vec{r}) \]

where \( s \) denotes the ray path.

The equation above can be given by following two coupled differential equations

\[ \begin{align*}
\frac{d\vec{r}}{ds} &= \frac{1}{n(\vec{r})} \vec{\nu} \\
\frac{d\vec{\nu}}{ds} &= \nabla n(\vec{r})
\end{align*} \]

\( \vec{\nu}_0 = (dx(t_0)/ds, dy(t_0)/ds, dz(t_0)/ds) = (\cos \alpha, \cos \beta, \sin(90^\circ - \gamma)) \)

\( \vec{r} = [x(t), y(t), z(t)] \) - point of trajectory, \( \vec{\nu} \) - tangent vector (to \( \vec{r} \)).

These coupled differential equations, that determine the ray propagation in a medium where the index of refraction as a function of position \( \vec{r} \) is given by, \( n(\vec{r}) \), can for example be solved using the Runge-Kutta technique.
Recurrent slant delay algorithm

Initial conditions
Coordinates of GPS station \((x_0, y_0, z_0)\)
Azimuth \(\psi_0\) and elevation \(\xi_0\)
Identification indexes of cell \((i, j, k)\): \((x_0, y_0, z_0) \in \text{cell}(i, j, k)\)

Recurrent algorithm
Refractivity approximation in cell \((i, j, k)\)
Integration of ray equation - new point of trajectory: \((x, y, z)\)
Identification indexes of cell \((i, j, k)\): \((x, y, z) \in \text{cell}(i, j, k)\)

Slant delay calculation
\[ \tau = \int (n - 1) \, ds \]
\(ds\) - calculated from points of trajectory \((x_k, y_k, z_k)\).
Model domains width 33 km and 13 km resolutions
Refractivity fields

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Evaluation slant delay functions

Scanning process for various elevation – $\xi$, $\phi$ - azimuth.
\( \xi = 3^0, 4^0, ..., 90^0, \ \phi = -180^0, ..., 180^0 \)

2d and 3d anisotropic distribution of slant delay \( \tau(\xi, \phi) \), \( \xi \) – elevation, \( \phi \) – azimuth.
\[ \tau_R(\xi, \phi) = \tau(\xi, \phi) - \overline{\tau(\xi, \phi)}, \quad \xi = 3^0, 4^0, \ldots, 90^0, \quad \phi = -180^0, \ldots, 180^0 \]

Reduced anisotropic distribution of slant delay.
2d and 3d anisotropic distribution of slant delay $\tau(\xi, \phi)$, $\xi$ – elevation, $\phi$ – azimuth
Reduced anisotropic distribution of slant delay
Module possibilities

Temporary and spatial analysis atmospheric refraction – GPS slant delay.