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Mesoscale mapping functions (Slant delay mesoscale functions)

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We present a prototype of computer module for GPS slant delay determination using data from COAMPS (Coupled Ocean/Atmosphere Mesoscale Prediction System) NRL (Naval Research Laboratory).

COAMPS is a mesoscale non-hydrostatic model of the atmosphere which is run on IA64 Feniks computer cluster in the Department of Civil Engineering and Geodesy of the Military University of Technology.

The slant delay is the result of integrating the ray (eikonal) equation for the spatial function of tropospheric refraction along the GPS wave propagation path. The work is a phase of research concerning the impact of mesoscale atmospheric phenomena on the tropospheric delay of the GPS signal.

Development (some facts)

- Neil A. E. (2001). Preliminary evaluation of atmospheric mapping functions based on numerical weather models. Physics an Chemistry of the Earth A(26), pp. 475-480.

- Boehm, J., A.E. Niell, P. Tregoning, H. Schuh (2006), Global Mapping Functions (GMF): A new empirical mapping function based on numerical weather model data, Geoph. Res. Letters, Vol. 33

- Boehm, J., B. Werl, and H. Schuh (2006), Troposphere mapping functions for GPS and very long baseline interferometry from European Centre for Medium-Range Weather Forecasts operational analysis data, J. Geophys. Res., 111.

- R. Eresmaa, H. Järvinen, An observation operator for Ground-based GPS slant delays, Tellus (2006), 58A, 131-140.

- R. Eresmaa, H. Järvinen, S. Niemelä, K. Salonen, Azimuthal asymmetry in ground-based GPS slant delay observations and their NWP model counterparts Atmospheric Chemistry and Physics Discussions, Vol. 7, pp 3179-3202, 27-2-2007

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Slant delay module

Mesoscale model - COAMPS

Subroutine: Refraction

Subroutine: Refraction approximation

Subroutine: Ray path and slant delay integration

Slant delay τ

$$\tau = \int (n-1)ds = 10^{-6} \int N ds$$

$$N = k_1 \frac{p_d}{T} Z_d^{-1} + k_2 \frac{e}{T} Z_w^{-1} + k_3 \frac{e}{T^2} Z_w^{-1}$$

n – refraction (refractive index), *N* – refractivity, p_d and *e* partial pressure of dry air and of water vapor, *T* – temperature, Z_d i Z_w – compressibility of dry air and of water vapor, k_1, k_2, k_3 constants determined experimetally.

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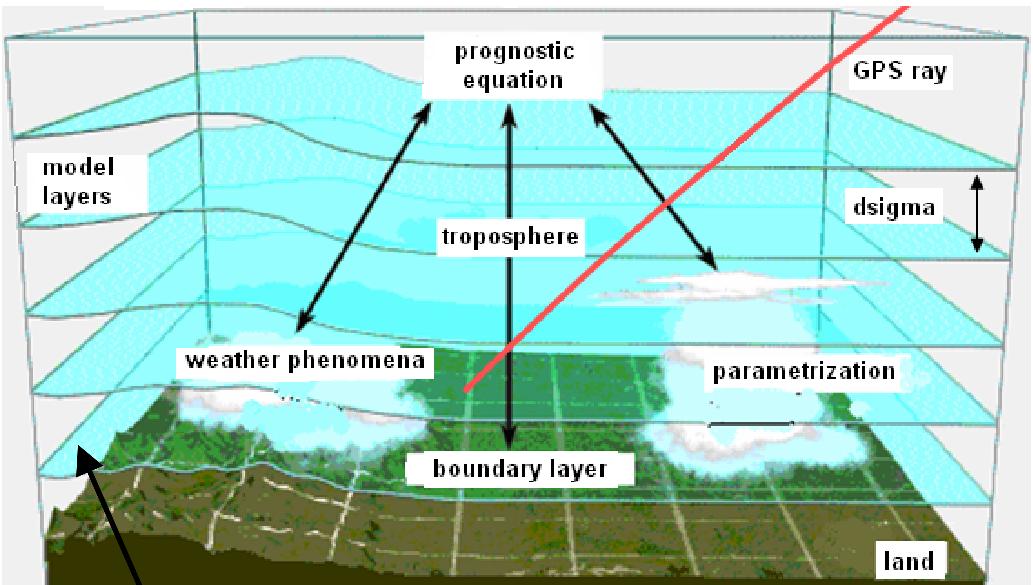
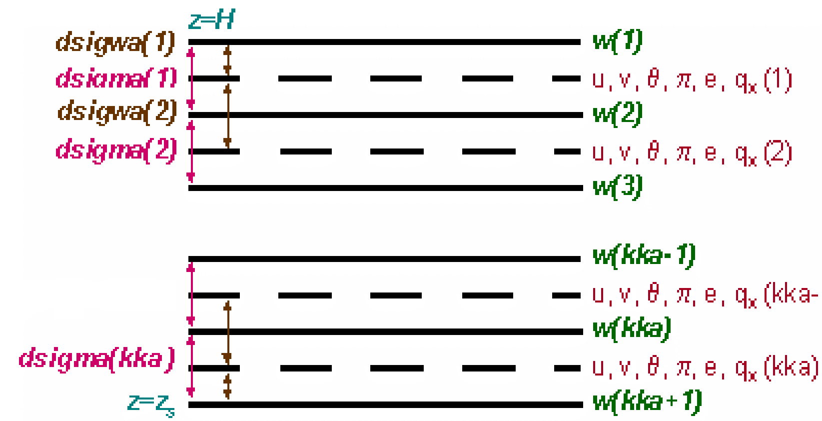
COAMPS

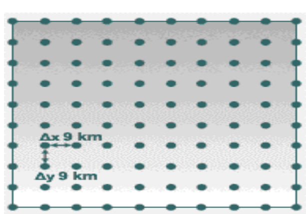
Mesoscale model domain



Atmospheric model vertical grids (Terrain following sigma Z system)



$$\sigma = z_{top}(z -_{s})/(z_{top} - z_{s})$$



z_{top} = H – depth of the model domain (31.50 km ~ 10 hPa),
z_s – height of topograhy, *kka* – number of levels (*kka* = 30).

Horizontal grids - Lambert Conformal projection .

Prognostic parameters:

π – Exner pressure (function),

θ – potential temperature,

q – specific humidity,

$\dot{u}$, v, w – velocity components

Diagnostic parameters:

T – temperature,

e – partial pressure of water vapor

p_d – partial pressure of dry air

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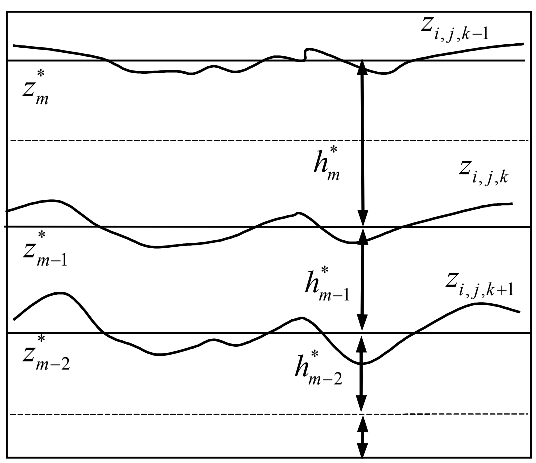
Refractivity approximation

$$N^*_{i,j,m} = N_{i,j,k\text{-}1} + w\cdot(N_{i,j,k} - N_{i,j,k\text{-}1}),\;\; w = (z^*_m - z_{i,j,k\text{-}1})/(z_{i,j,k} - z_{i,j,k\text{-}1})$$

Linear aproximation



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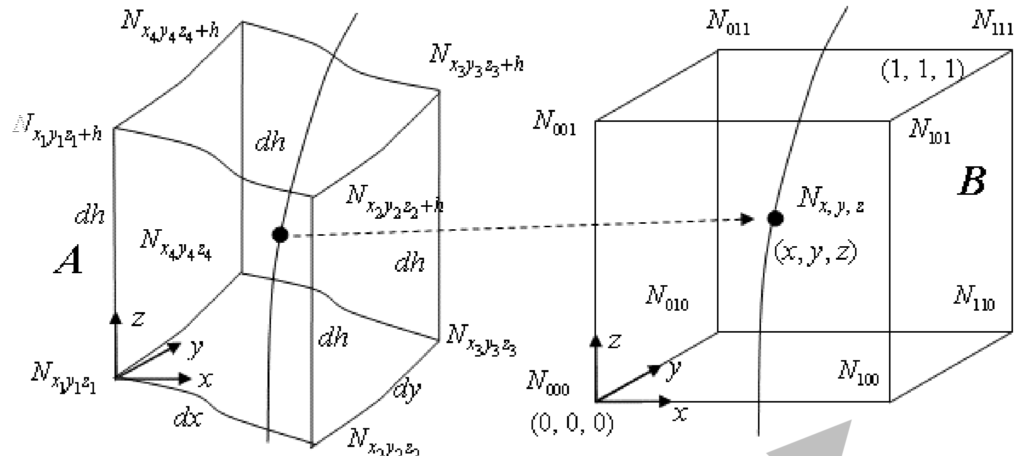


Trilinear interpolation

$$N_{xyz} = N_{000}(1-x)(1-y)(1-z) + N_{100}x(1-y)(1-z) + N_{010}(1-x)y(1-z) + N_{001}(1-x)(1-y)z +$$
$$N_{101}x(1-y)z + N_{011}(1-x)yz + n_{110}xy(1-z) + N_{111}xyz$$
$$x = x/dx,\; y = y/dy,\;\; z = z/dh^*$$

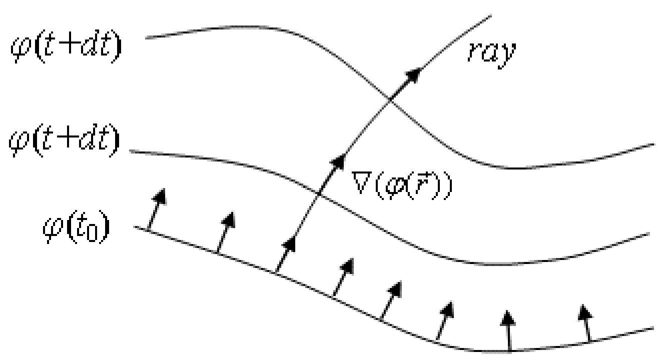
Polynomial structure of *N* makes easy gradient ∇N calculation

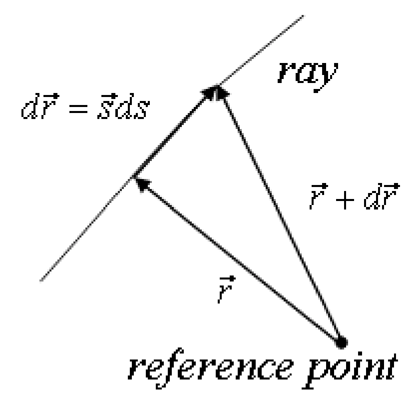
Ray tracing in the atmosphere. Ray definition.

Propagation of rays are determined by the eikonal equation:

$$\sum_{i=1}^{k} \left(\frac{\partial \varphi(\vec{r})}{\partial r_i}\right)^2 = n^2(\vec{r}), \;\; k = 3$$

where the gradient of $\varphi(\vec{r})$ gives the direction of the ray, $n(\vec{r})$ index of atmospheric refraction. The $\varphi(\vec{r})$ function is often called eikonal.





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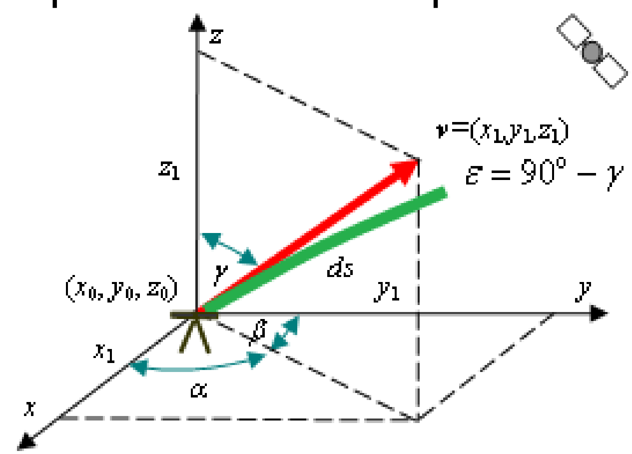
The ray equation is the eikonal equation in a new ray based coordinate

$$\frac{d}{ds}\left[n(\vec{r})\frac{d\vec{r}}{ds}\right] = \nabla n(\vec{r})$$

where *s* denotes the ray path.
The equatio nabove can be given by following two coupled differential equations

$$\begin{cases} \dfrac{d\vec{r}}{ds} = \dfrac{1}{n(\vec{r})}\vec{v} \\[2ex] \dfrac{d\vec{v}}{ds} = \nabla n(\vec{r}) \end{cases}$$

$$\vec{v}_0 = (dx(t_0)/ds, dy(t_0)/ds, dz(t_0)/ds) = (\cos\alpha,\ \cos\beta, \sin(90^\circ - \gamma))$$



$\vec{r} = [x(t), y(t), z(t)]$ - point of trajectory ,$\vec{v}$ - tangent vector (to $\vec{r}$).

These coupled differential equations, that determine the ray propagation in a medium where the index of refraction as a function of position $\vec{r}$ is given by, $n(\vec{r})$, can for example be solved using the Runge-Kutta technique.

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Recurrent slant delay algorithm

Initial conditions

Coordinates of GPS station (x_0, y_0, z_0)

Azimuth ψ_0 and elevation ξ_0

Identification indexes of cell(i,j,k): $(x_0, y_0, z_0) \in$ cell(i,j,k)

Recurrent algorithm

Refractivity approximation in cell(i,j,k)

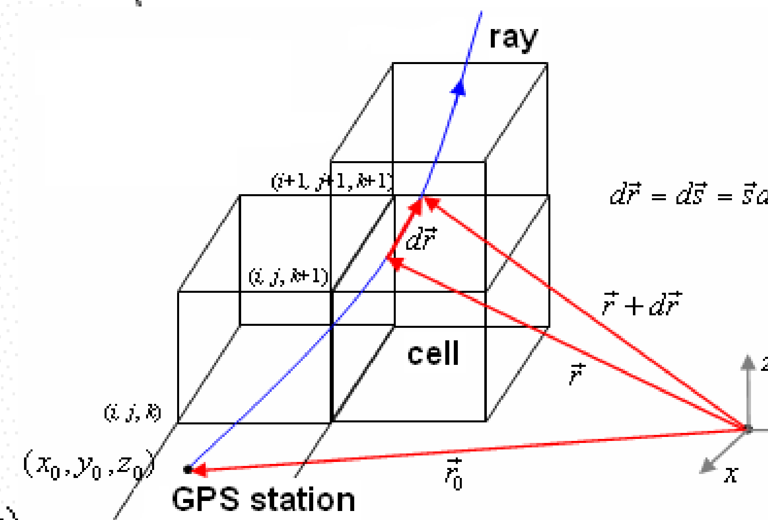
Integration of ray equation - new point of trajectory: (x, y, z)

Identification indexes of cell(i,j,k): $(x, y, z) \in$ cell(i,j,k)

Slant delay calculation

$$\tau = \int (n-1) ds$$

ds - calculated from points of trajectory (x_k, y_k, z_k) .



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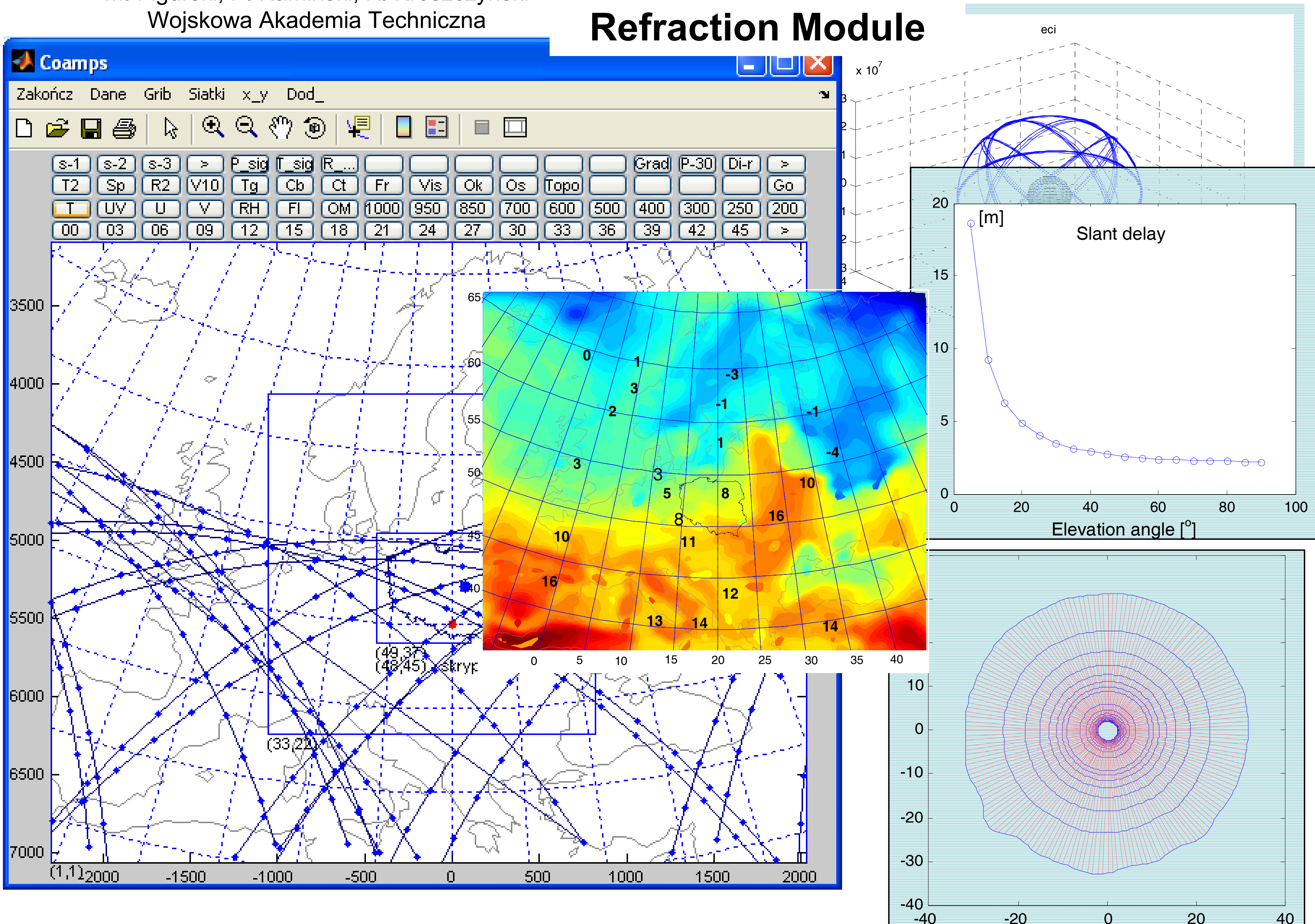
Refraction Module



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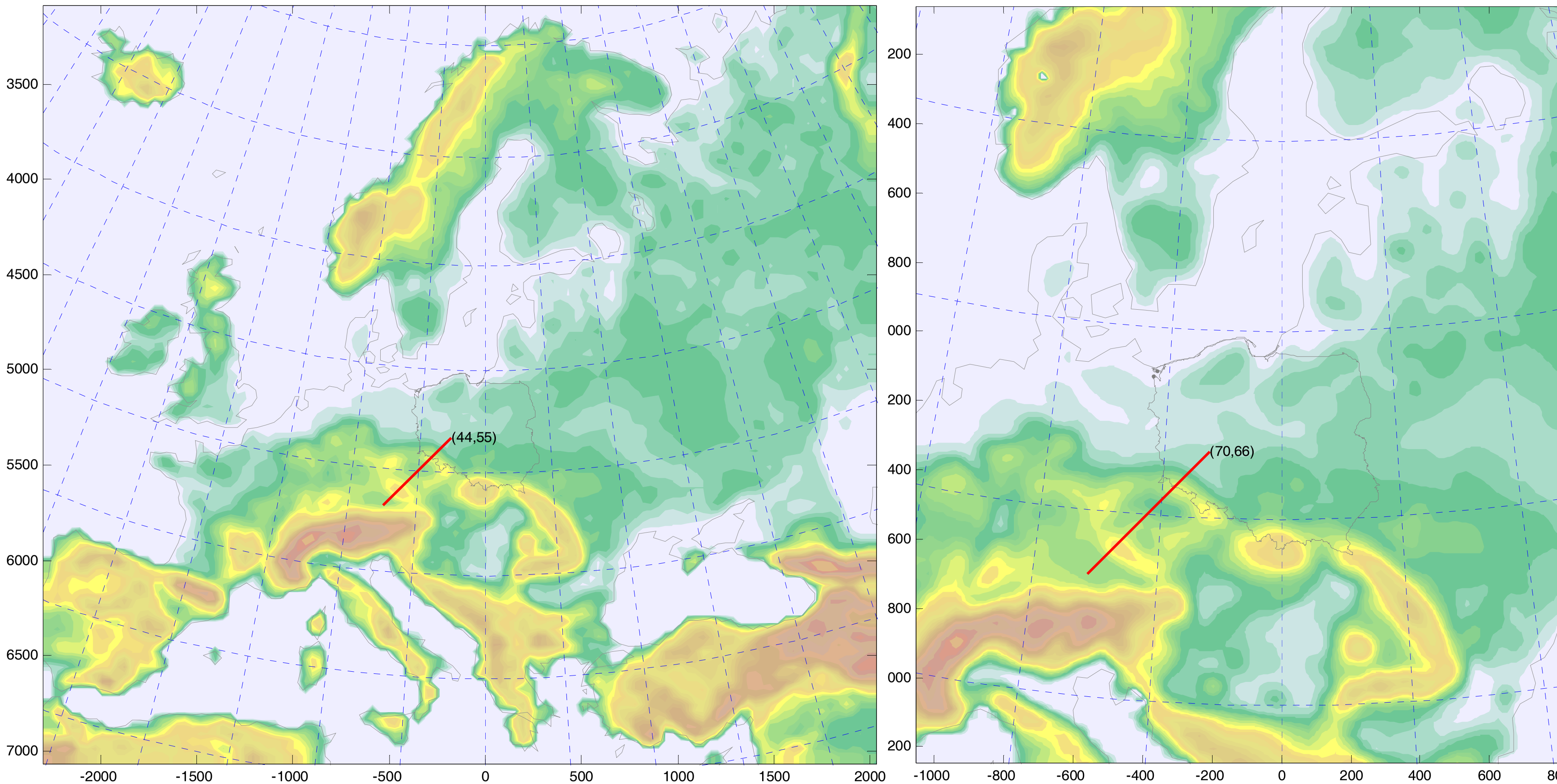

Model domains width 33 km and 13 km resolutions

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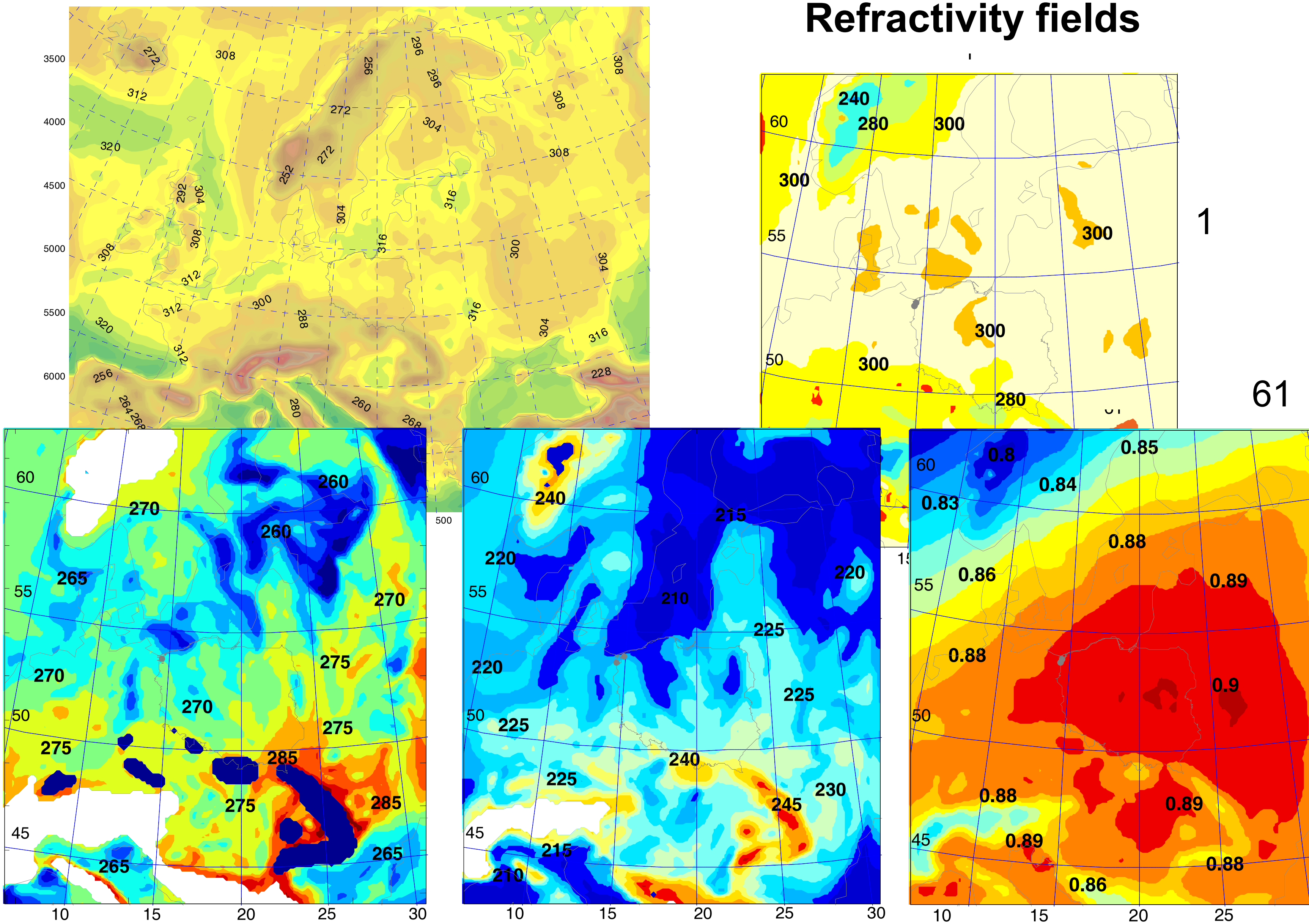
Refractivity fields



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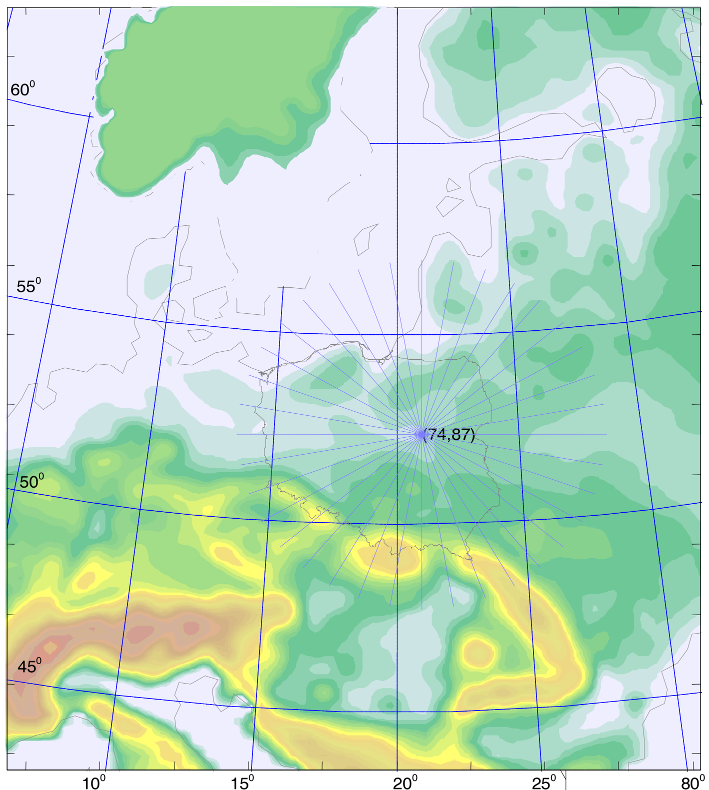
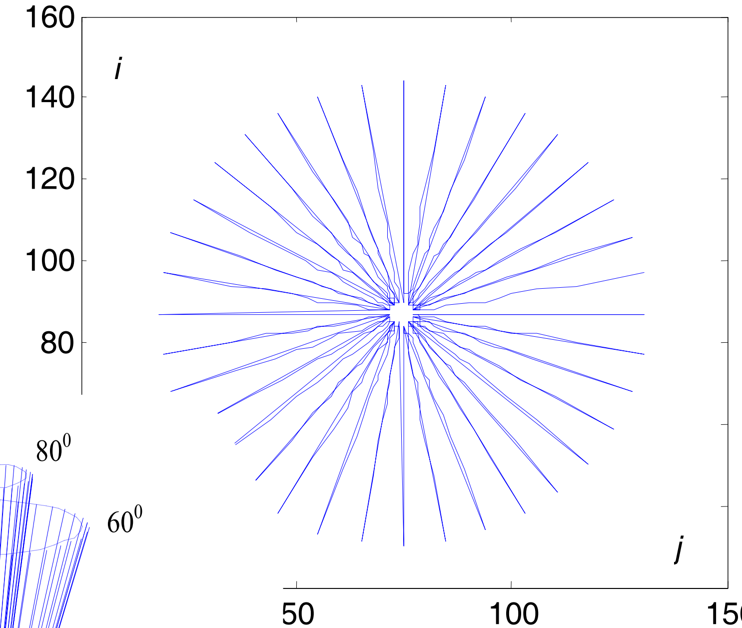
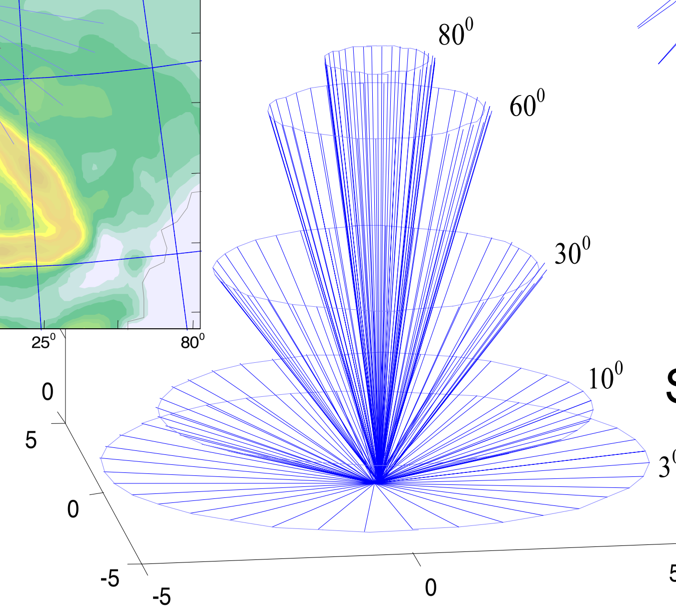
Evaluation slant delay functions



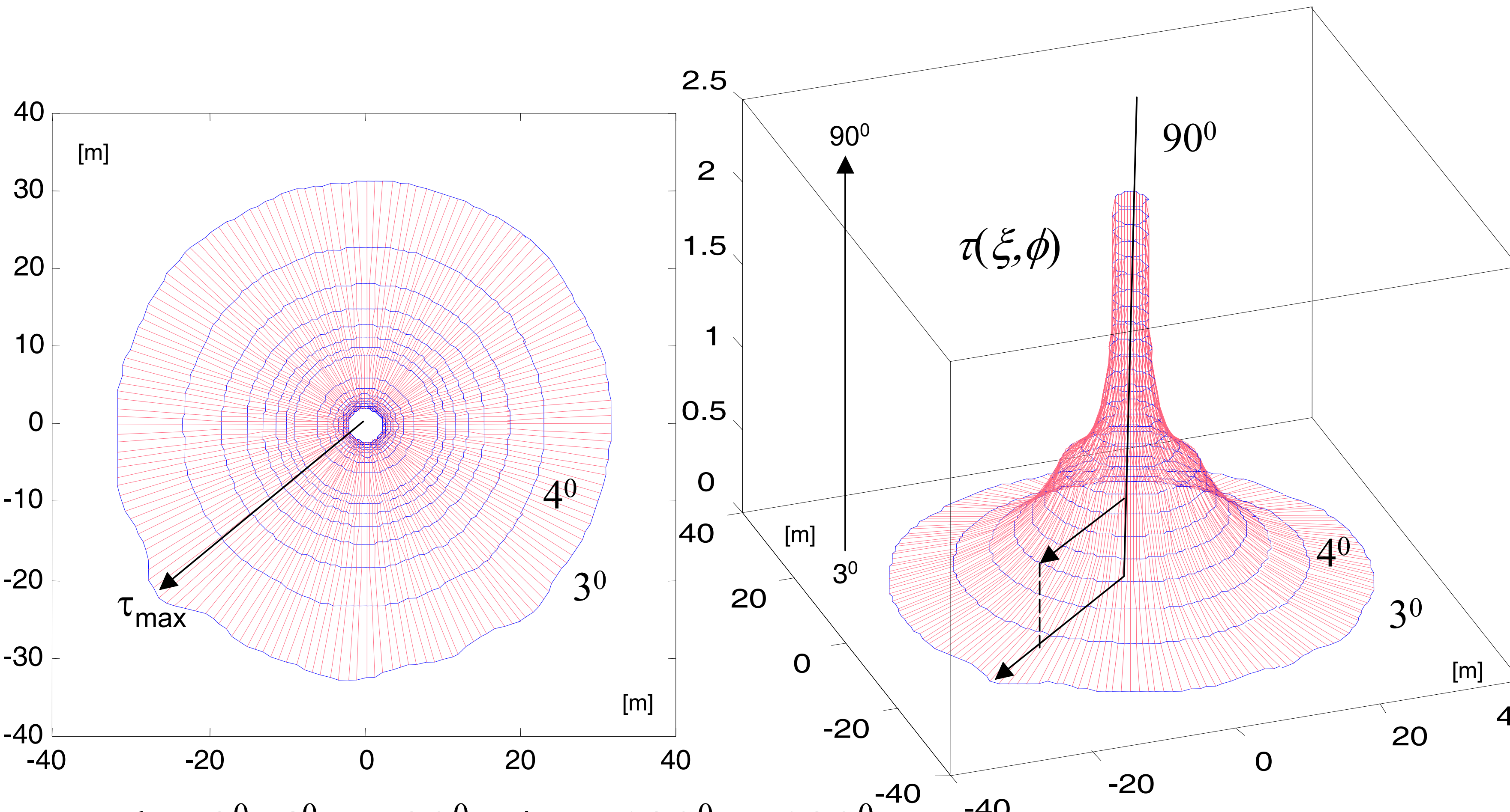
Reference point



Scaning domain



Scaning process for various elevation – ξ, ϕ - azimuth.



$$\xi = 3^0, 4^0, \ldots, 90^0,\ \phi = -180^0, .., 180^0$$

2d and 3d anisotropic distribution of slant delay $\tau(\xi,\phi)$, ξ – elevation, ϕ – azimuth.

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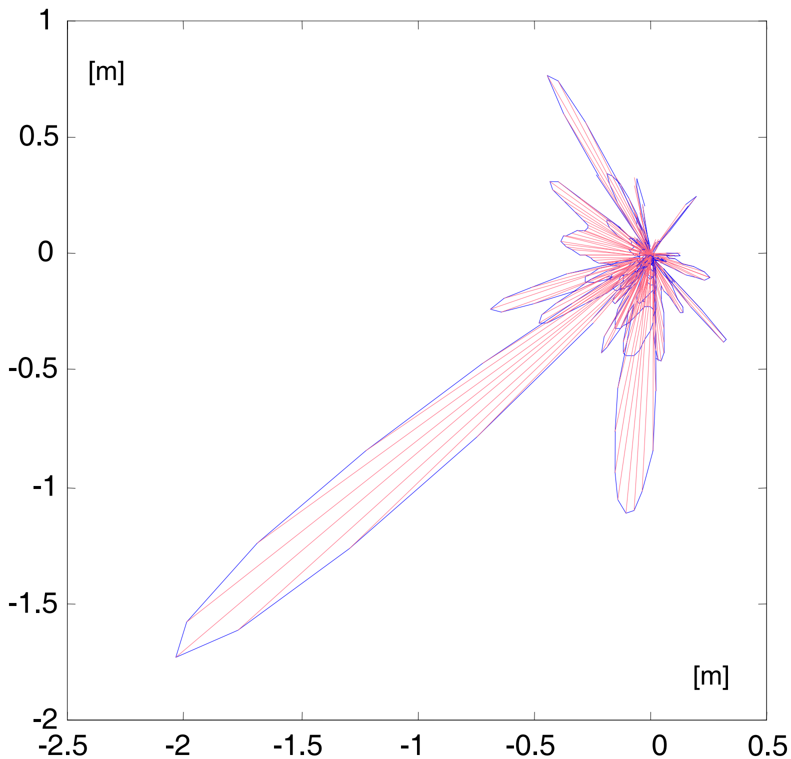
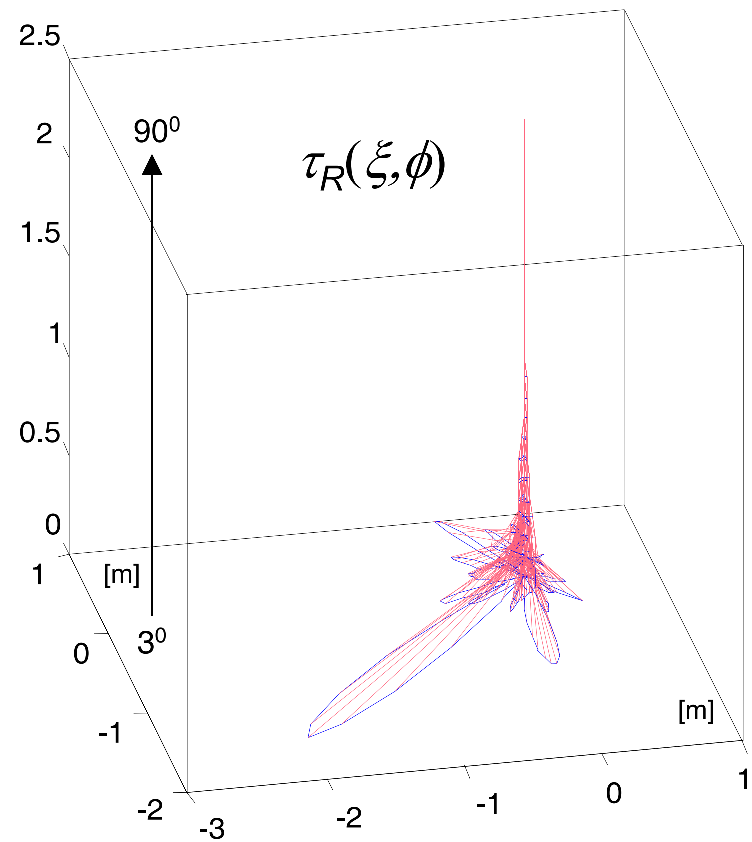
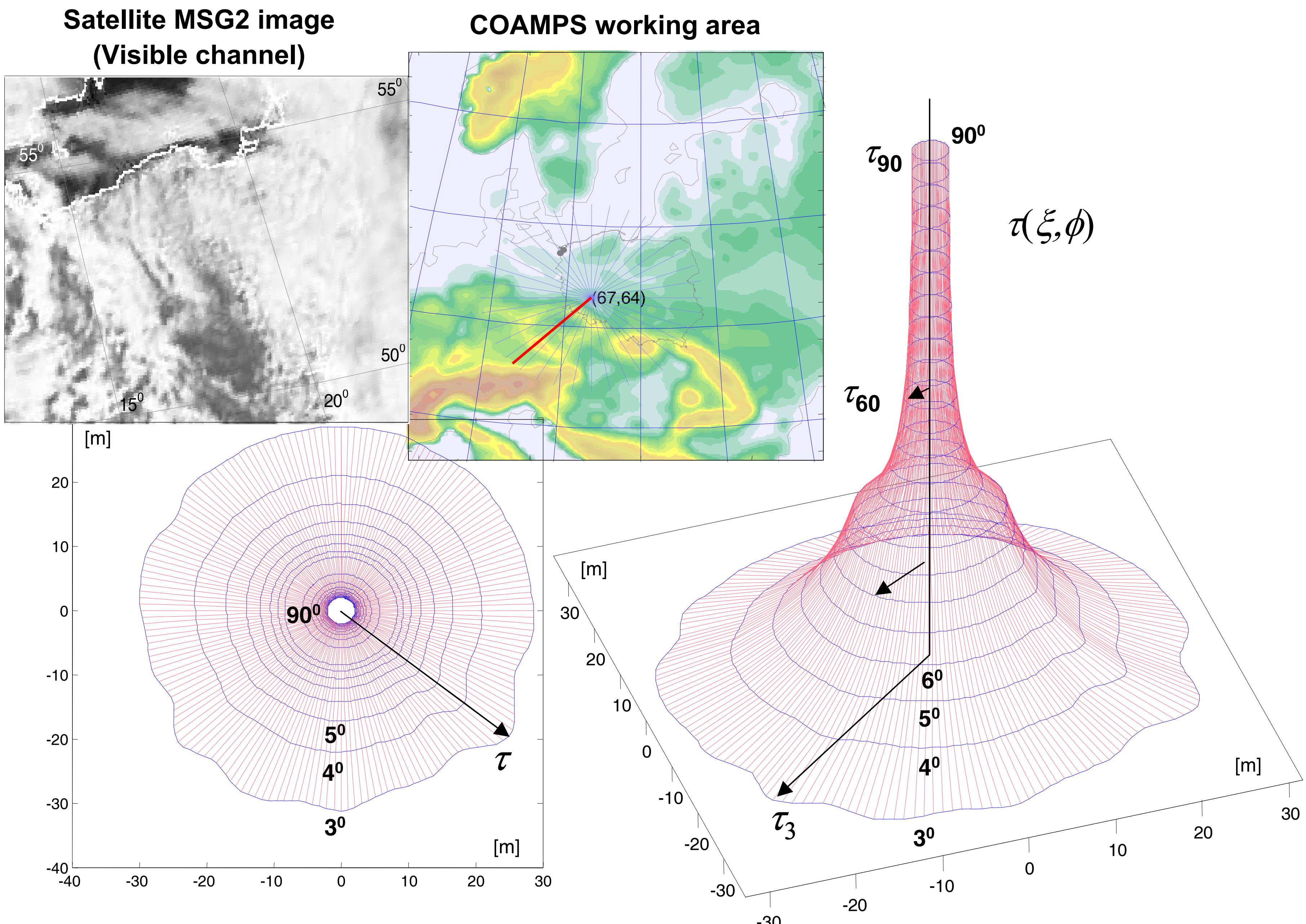


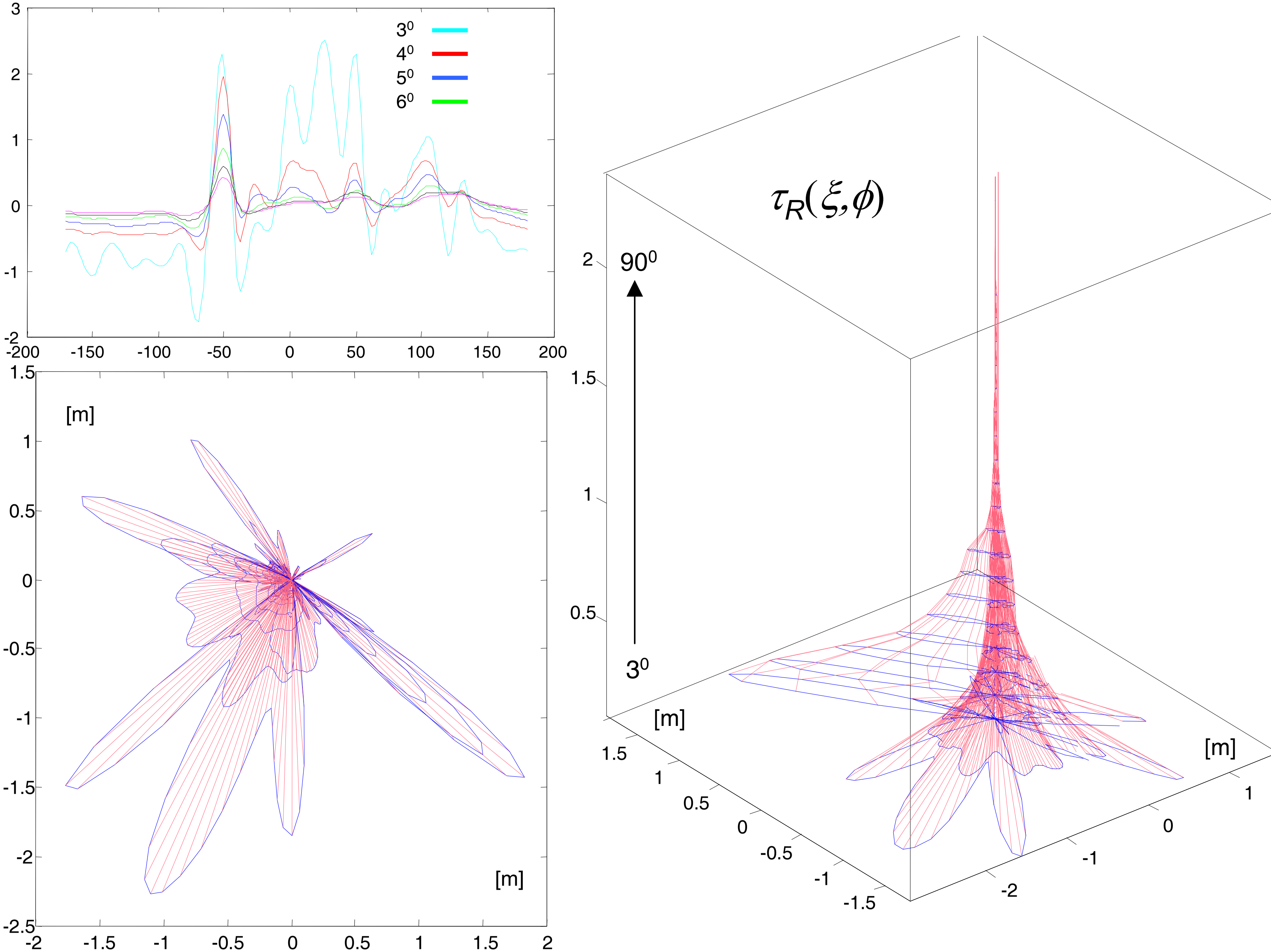


$$\tau_R(\xi,\phi) = \tau(\xi,\phi) - \overline{\tau(\xi,\phi)},\ \ \xi = 3^0, 4^0, \ldots, 90^0,\ \phi = -180^0, .., 180^0$$

Reduced anisotropic distribution of slant delay



2d and 3d anisotropic distribution of slant delay $\tau(\xi,\phi)$, ξ – elevation, ϕ –azimuth



Reduced anisotropic distribution of slant delay

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Module possibilities

Temporary and spatial analysis atmospheric refraction – GPS slant delay.