ALGORITHM OF GPS BASELINE PROCESSING IN REAL TIME MODE FOR ENGINEERING SURVEYS

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Abstract: A detailed algorithm of GPS baseline processing using sequential adjustment is presented in this paper. This algorithm offers opportunity to detect the changes of point position instantly. Therefore it can be used in automated deformation monitoring systems. The new software was created on the basis of presented algorithm. This software is expected to serve mainly for research purposes.

1. Introduction

The automated, monitoring deformation systems necessitate implementation of real time mode baseline processing algorithm. This algorithm must ensure fast calculations and oportunity of instant detecting the changes of point position. Sequential adjustment algorithm has these features. The way of using the sequential adjustment algorithm to baseline processing in real time mode is described below.

2. Observation equations

In proposed algorithm two types of observation are used: the pseudoranges and carrier phases. We can show the observation equations for both types of observations as follow[1],[2],[3],[4]:

$$\Phi=\tfrac{f}{c}\rho+f(dt^{s}-dt_{r})+\tfrac{f}{c}\delta^{trop}-\tfrac{f}{c}\delta^{jon}+N+\varepsilon_{\Phi} \quad (1)$$

$$P=\rho+c(dt^{s}-dt_{r})+\delta^{trop}+\delta^{jon}+\varepsilon_{P}$$

where:

Φ - measured carrier phase

f - carrier frequency

ρ - geometric range receiver-satellite

c - vacuum speed of light

dt^s - offset of satellite clock

dt_r - offset of receiver clock

δ^{trop}, δ^{jon} -delays due to the troposphere and ionosphere

ε_{Φ}, ε_p - the effect of measurement noise for carrier phases and pseudoranges respectively

In proposed algorithm double differenced carrier phase and pseudoranges are used:

$$\nabla\Delta\Phi=\nabla\Delta\rho+\nabla\Delta N+\nabla\Delta\varepsilon_{\Phi}$$

$$\nabla\Delta P=\nabla\Delta\rho+\nabla\Delta\varepsilon_{P} \qquad (2)$$

It is assumed that the clock offsets and the effect of ionosphere are removed by double differencing the observations.

3. Ambiguity resolution

To resolve ambiguities both types of observation: pseudoranges and carrier phases are used.

Therefore a preliminary adjustment is performed. In this adjustment following functional model is used:

$$\underline{V}=\underline{A}\underline{X}+\underline{L}\ , \qquad (3)$$

where:

$\underline{V}$ - corrections vector

$\underline{A}=\begin{bmatrix} A_P \\ A_{DD} \end{bmatrix}$ **A_p, A_{DD} - functional model matrices for double differenced pseudoranges and carrier phases respectively**

$\underline{X}=\begin{bmatrix} X_c \\ X_a \end{bmatrix}$ **X_c-coordinates increment vector, X_a - ambiguity vector**

$\underline{L}=\begin{bmatrix} L_p \\ L_{DD} \end{bmatrix}$ **L_p, L_{DD} - free terms vectors for double differenced pseudoranges and carrier phases respectively**

Statistical model can be written as:

$$\underline{C}=\delta^2\underline{Q} \tag{4}$$

where:

$\underline{C}$ - covariance matrix

$\delta^2 = \frac{V^T C^{-1} V}{n-m}$ **- variance coefficient**

(n - number of observations, m- number of parameters

$\underline{Q} = \begin{bmatrix} Q_p & \\ & Q_{DD} \end{bmatrix}$ **Q_p, Q_{DD} -cofactor matrices for pseudoranges and double differenced carrier phases respectively**

Hence the solution of least squares estimation is the following vector:

$$\underline{X}=(\underline{A}^T\underline{C}^{-1}\underline{A})^{-1}\underline{A}^T\underline{C}^{-1}\underline{L} \tag{5}$$

and his variance matrix:

$$C_X= \delta^2 (A^T C^{-1} A)^{-1} \tag{6}$$

Matrix C_X has following structure:

$C_X = \begin{bmatrix} C_c & \\ & C_a \end{bmatrix}$**, C_c, C_a - covariance matrices of coordinates and ambiguites respectively.**

Algorithm of preliminary adjustment can be shown as follow:

1. **Acquisition of observation data (pseudorange and phases) from the first epoch**
2. **Adjustement**
3. **Testing the following condition:**

   $$\text{maximum}(\text{diag}(C_a))< \sigma^2_{max} \tag{7}$$

   where σ^2_{max} is constant
4. **If condition (7) returns false then number of observations is increased by adding observation set from next epoch in next adjustment.**
5. **When condition (7) returns true the preliminary adjustment is finished. Integer ambiguities are calculeted from last step using vector X_a and C_a matrix with LAMBDA method [5].**
6. **Finally adjustment of the double differenced carrier phases with fixed, integer ambiguites is performed. The functional model of this adjustment can be presented by means of the following system of the correction equations:**

$$\underline{V}_{DD}=\underline{A}_{DDc}\underline{X}_c+\underline{L}_{DDc} \tag{8}$$

where:

$\underline{L}_{DDc}=\underline{L}_{DD}+\underline{A}_{DDa}\underline{X}_{afix}$

$\underline{A}_{DDc}$, $\underline{A}_{DDa}$ - submatrices of A_{DD} refering to coordinates and ambiguites respectively

$\underline{X}_{afix}$ - vector of fixed, integer ambiguites

4. Sequential adjustment

Functional model for the sequential adjustment reads as follow:

$$\underline{V}_s=\underline{A}_s\underline{X}_c+\underline{L}_s \tag{9}$$

where:

$\underline{V}_s$ - residuals vector

$\underline{A}_s = \begin{bmatrix} E \\ A_{DDc} \end{bmatrix}$ **E is 3x3 dimension unit matrix**

$\underline{L}_s = \begin{bmatrix} 0 \\ L_{DDc} \end{bmatrix}$ **0 is 3x1 dimension vector of zeros**

Statistical model can be presented in the form of following covariance matrix:

$$\underline{C}_s=\delta^2\underline{Q}_s$$
(10)

where:

$\underline{Q} = \begin{bmatrix} Q_c & \\ & Q_{DD} \end{bmatrix}$, **$Q_c$ - cofactor matrix for coordinates**

If preliminary adjustment observations set consists of observations taken from n epochs, then sequential adjustment starts with n+1st epoch (the n+1st epoch of baseline processing is the first epoch of sequential adjustment). In each successive epoch separate adjustment is performed. The matrix $\underline{A}_s$ and the vector $\underline{L}_s$ are formed on the basis of coordinates obtained from previous adjustment and carrier phases from present epoch. In the first epoch of sequential adjustment elements of matrix A_{DDc} and vector L_{DDc} are determined on the basis of coordinates taken from preliminary adjustment and fixed ambiguites.

5. Cycle slips detection

Before matrix $\underline{A}_s$ and vector $\underline{L}_s$ are formed, cycle slips must be detected. Solution of this problem is based on triple differenced carrier phases analysis. Triple differences are formed as differences of double differenced carrier phases from last three successive epochs. This values (for each pair of satellites) are stored and updated in two-elements vectors. It is assumed that cycle slip appears if rounded difference of that two elements differs from zero. If cycle slip is detected the appropriate value of ambiguity is changed.

Assume dd(i), i=1, 2, .. 5 as a time series for the double differenced carrier phases which contain cycle slip of x at 4-th epoch:

i (epoch)	dd(i)	td	qd
1	dd(1)		
		td_1	
2	dd(2)		qd_1
		td_2	
3	dd(3)		qd_2+x
		td_3+x	
4	dd(4)+x		qd_3-x
		td_4	
5	dd(5)+x		

In this scheme the third column contains triple differences of carrier phases and fourth column the differences of the triple differences (quadruple differences). The values of elements from the fourth column are small (usually below 0.1). Therefore the rounded values of fourth column elements equals values of cycle slips (apart of next after non zero element – it must be corrected by adding the value of cycle slip). In fact if in i-th epoch a cycle slip appears then in i+1st epoch the value of cycle slip is added to 1-st element (from i-1st epoch) of double difference vector. In this way the triple differences vector and the quadruple difference in i+1st epoch are free from effect evoked by cycle slip from i-th epoch.

If there are no cycle slips the values of quadruple differences equals zero.

The values of double differences, triple differences, and quadruple difference in fourth epoch (when appears a cycle slip) are inside the triangle on the scheme.

6. Results of the test

The algorithm was applied to raw GPS data. The results of preliminary adjustments are given in Fig. 1. Mean errors of ambiguities were calculated as square roots of diagonal elements $\underline{C}_a$ from formula (6). In the test it took 6 epochs (with interval=20 sec.) of observations before the mean errors of ambiguities were lower than σ_{max}=0.3.

Differences of parameter values in successive epochs and their mean errors are presented on Fig. 2. The results in 6-th epoch were obtained from adjustment of data set consists of double differenced carrier phases from epoch: 1 to 6 with fixed ambiguities.

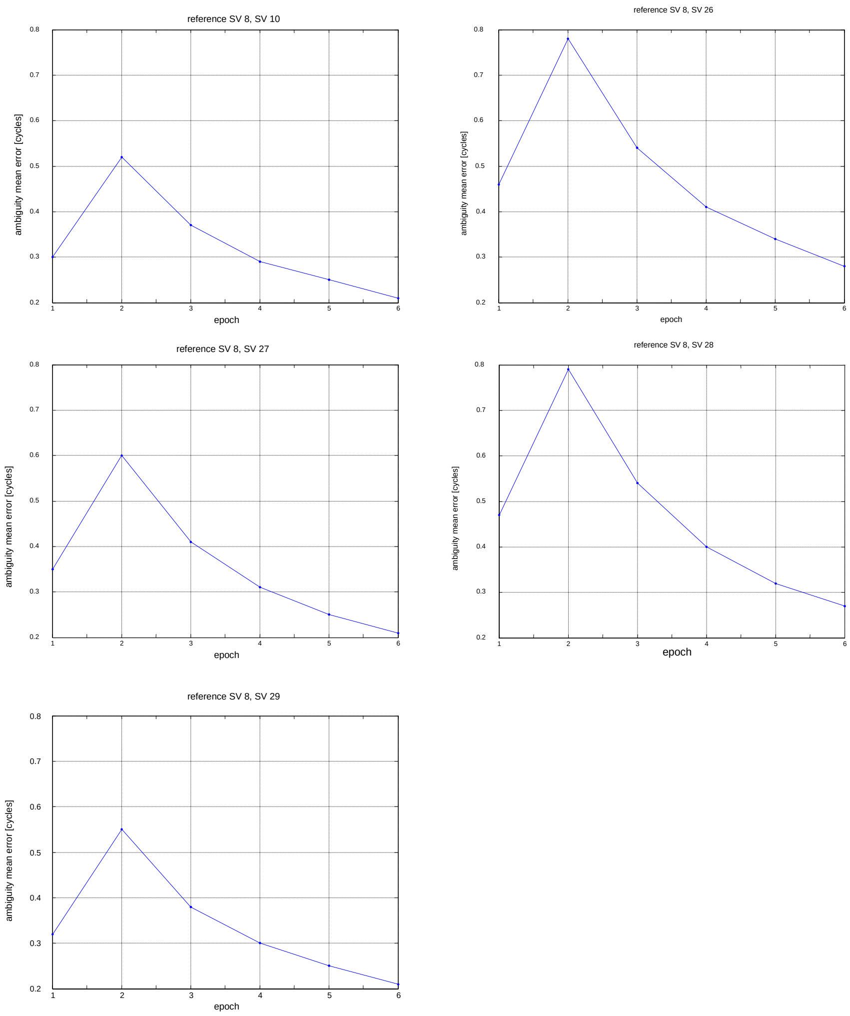

Fig. 1 Mean errors of ambiguities

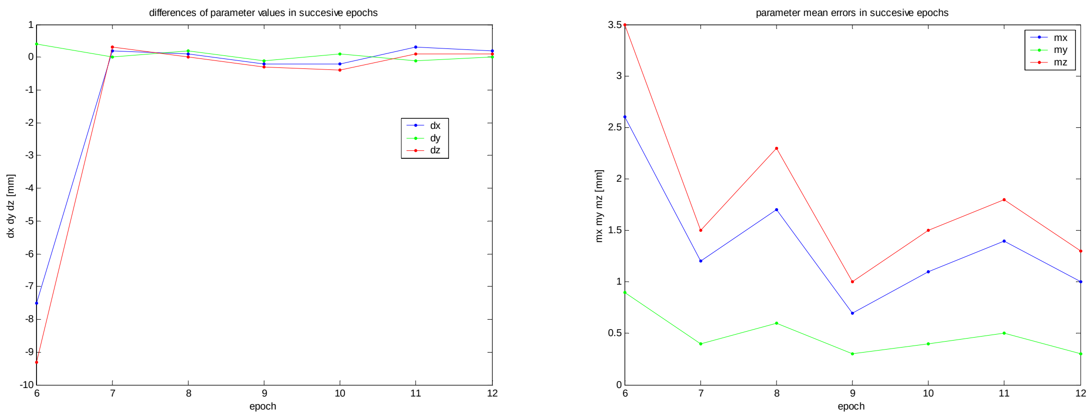

Fig. 2 Differences of parameter values in successive epochs and their mean errors

Subsequent results derive from sequential adjustment. Starting from 7-th epoch the maximal changes of coordinates in successive epochs are lower than 1 mm. Mean errors do not exceed 3 mm.

epoch	data	ref SV 8, SV 10	ref SV 8, SV 26	ref SV 8, SV 27	ref SV 8, SV 28	ref SV 8, SV 29
12	double differences	67382.677 67379.221 67375.766	-223923.791 -223925.038 -223926.291	100820.867 100819.956 100819.001	-161858.912 -161859.286 -161859.657	-148874.171 -148875.591 -148876.995
	triple differences	-3.456 -3.455	-1.247 -1.253	-0.911 -0.955	-0.374 -0.371	-1.420 -1.404
	quadruple differences	0.001	-0.006	-0.044	0.003	0.016
	cycle slips	0	0	0	0	0
13	double differences	67379.221 67375.766 67372.317	-223925.038 -223926.291 -223927.539	100819.956 100819.001 100818.043	-161859.286 -161859.657 -161860.011	-148875.591 -148876.995 -148878.401
	triple differences	-3.455 -3.449	-1.253 -1.248	-0.955 -0.958	-0.371 -0.354	-1.404 -1.406
	quadruple differences	0.006	0.005	-0.003	0.017	-0.002
	cycle slips	0	0	0	0	0
14	double differences	67375.766 67372.317 67368.840	-223926.291 -223927.539 -223928.784	100819.001 100818.043 100817.070	-161859.657 -161860.011 -161860.414	-148876.995 -148878.401 -148871.871
	triple differences	-3.449 -3.477	-1.248 -1.245	-0.958 -0.973	-0.354 -0.403	-1.406 6.530
	quadruple differences	-0.028	0.003	-0.015	-0.049	7.936
	cycle slips	0	0	0	0	8
15	double differences	67372.317 67368.840 67365.404	-223927.539 -223928.784 -223930.009	100818.043 100817.070 100816.097	-161860.011 -161860.414 -161860.760	-148870.401 -148871.871 -148873.287
	triple differences	-3.477 -3.436	-1.245 -1.225	-0.973 -0.973	-0.403 -0.346	-1.470 -1.416
	quadruple differences	0.041	0.020	0.000	0.057	0.054
	cycle slips	0	0	0	0	0

Tab. 1 Cycle slips detection

Tab. 1 includes the quantities that were used as basis for cycle slips detection. In presented example the cycle slip appears in 14-th epoch for following pair of satellites: 8, 29. Its value (8 cycles) was determined as rounded value of quadruple difference (7.936).

7. Final remarks

Algorithm described in this paper gives reliable results on the basis of the data set from several epochs and offers the possibility of detecting instant changes in point position.

References:

[1] Hoffman-Wellenhof B., Lichtenegger H., Collins J., *Global Positioning System-Theory and Practice,* fourth edition., Springer-Verlag, Wien, New York, 1997.

[2] Leick A. *GPS Satellite Surveing,* Publisher John Wiley & Sons Inc., USA, 1990

[3] Rzepecka Z., *Modification of algorithms for determination of short kinematic vectors from GPS data,* rozprawa habilitacyjna, wydawnictwo UWM Olsztyn, 2004

[4] Strang G., Borre K. *Linear Algebra, Geodesy and GPS.* Wellesley-Cambridge Press, 1997.

[5] Teunissen P. *The least-squares ambiguity decorrelation adjustment: a method for fast GPS integer ambiguity estimation.* Journal of Geodesy 70, 1-2 (1995), 65-82