Attenuation Matrix in Robust, Free Adjustment

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Presentation layout

- Free Adjustment
- Covariance matrices
- Robustness in Free Adjustment
- Attenuation Matrix
- Example

Free Adjustment

- Residual equation - $\mathbf{v} = \mathbf{A}\mathbf{d}_{\mathbf{X}} + \mathbf{l}$
- Solution - $\hat{\mathbf{d}}_{\mathrm{X}} = -\mathbf{A}^{+}_{\mathbf{PP}_{\mathrm{X}}}\mathrm{l}$
- Properties - $\hat{\mathbf{d}}^{T}_{\mathrm{X}}\mathbf{P}_{\mathrm{X}}\hat{\mathbf{d}}_{\mathrm{X}} = \min$

$$\left(\mathbf{A}\hat{\mathbf{d}}_{\mathbf{X}} + \mathbf{l}\right)^{T}\mathbf{P}\left(\mathbf{A}\hat{\mathbf{d}}_{\mathbf{X}} + \mathbf{l}\right) = \hat{\mathbf{v}}^{T}\mathbf{P}\hat{\mathbf{v}} = \min$$

Covariance Matrices

- Models - $\mathbf{C}_{x} = \sigma_0^2 \mathbf{P}^{-1}$ $\mathbf{C}_{\mathbf{X}} = \sigma_{0\mathbf{X}}^2 \mathbf{P}_{\mathbf{X}}^{-1}$
- Propagation - $\left(\mathrm{C}_x, \mathrm{C}_{\mathrm{X}}\right) \Rightarrow \mathrm{C}_1 \Rightarrow \mathrm{C}_{\hat{\mathrm{d}}_{\mathrm{X}}}$
- Solution - $\mathbf{C}_{\hat{\mathbf{d}}_{\mathbf{X}}} = \sigma_{0\mathbf{X}}^2 \mathbf{A}_{\mathbf{PP}_{\mathbf{X}}}^{+} \mathbf{A}^T \mathbf{P}_{\mathbf{X}}^{-1} \mathbf{A} \left(\mathbf{A}_{\mathbf{PP}_{\mathbf{X}}}^{+}\right)^T +$

$$+ \sigma_0^2 \mathbf{A}_{\mathbf{PP}_{\mathbf{X}}}^{+} \mathbf{P}^{-1} \left(\mathbf{A}_{\mathbf{PP}_{\mathbf{X}}}^{+}\right)^T$$

Robustness in Free Adjustment

- Equivalent Weight Matrix $\widehat{\mathbf{P}} = \mathbf{T}(\overline{\mathbf{v}})\mathbf{P}$

- Equivalent Weight Matrix for Approximate Coordinates $\widehat{\mathbf{P}}_X = \mathbf{T}(\overline{\mathbf{d}}_X)\mathbf{P}_X$

- Attenuation function $t(\overline{d}_X), \quad \overline{d}_X = \overline{d}_{X1}, \overline{d}_{X2}, \ldots, \overline{d}_{Xu}$

$$\left.\begin{array}{llll} i) & t(\overline{d}_{\mathbf{X}}) = 1 & for & \overline{d}_{\mathbf{X}} \in \Delta_X = \langle -k_X, k_X \rangle \\ ii) & t(\overline{d}_{\mathbf{X}}) < 1 & for & \overline{d}_{\mathbf{X}} \notin \Delta_X \\ \multicolumn{4}{l}{iii) \ \forall\ \overline{d}_{\mathbf{X}i}, \overline{d}_{\mathbf{X}j} \notin \Delta_X, |\overline{d}_{\mathbf{X}i}| < |\overline{d}_{\mathbf{X}i}| : t(\overline{d}_{\mathbf{X}i}) > t(\overline{d}_{\mathbf{X}j})} \end{array}\right\}$$

Robust, Free Adjustment

- Example
$$t(\bar{d}_{\mathbf{x}}) = \begin{cases} 1 & \text{for} \quad \bar{d}_{\mathbf{x}} \in \Delta_X \\ \exp\left[-l(\bar{d}_{\mathbf{x}} - k_X)^g\right] & \text{for} \quad \bar{d}_{\mathbf{x}} \notin \Delta_X \end{cases}$$

- Covariance Matrix of Increments

$$\mathrm{C}_{\hat{\mathrm{d}}_\mathrm{X}} = \mathrm{C}_{\hat{\mathrm{d}}_\mathrm{X}/\mathrm{X}^\circ} + \mathrm{C}_{\hat{\mathrm{d}}_\mathrm{X}/x} = \sigma_{0\mathrm{X}}^2 \mathrm{Q}_{\hat{\mathrm{d}}_\mathrm{X}/\mathrm{X}^\circ} + \sigma_0^2 \mathrm{Q}_{\hat{\mathrm{d}}_\mathrm{X}/x} =$$
$$= \sigma_0^2 \left(r_\sigma \mathrm{Q}_{\hat{\mathrm{d}}_\mathrm{X}/\mathrm{X}^\circ} + \mathrm{Q}_{\hat{\mathrm{d}}_\mathrm{X}/x} \right)$$

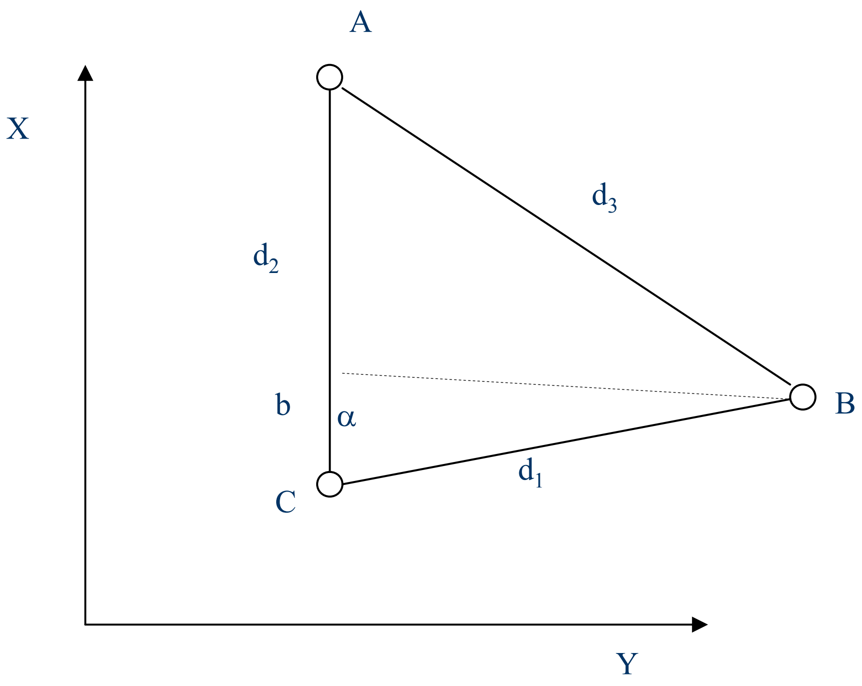
Robust, Free Adjustement

- Solution - $$\hat{\mathbf{d}}_X = -\mathbf{A}^+_{P\widehat{P}_X}\mathbf{l}$$

- Iterative Process -

$$\left.\begin{aligned}
\widehat{P}_X &= \lim_{j\to\infty} P_X^j \\
\hat{d}_X &= \lim_{j\to\infty} d_X^j \\
d_X^j &= -\left(A^+_{PP_X}\right)^j l = -\left(P_X^j\right)^{-1} B^T \left[B\left(P_X^j\right)^{-1} B^T\right]^{-1} A_1^T P l \\
P_X^j &= T\left(\bar{d}_X^{j-1}\right) P_X^{j-1}
\end{aligned}\right\}$$

Example



$$\mathbf{X}^\circ = \begin{bmatrix} X_A \\ Y_A \\ X_B \\ Y_B \\ X_C \\ Y_C \end{bmatrix} = \begin{bmatrix} 200.00\,m \\ 100.00\,\boldsymbol{m} \\ 100.00\,\boldsymbol{m} \\ 200.00\,\boldsymbol{m} \\ 100.00\,\boldsymbol{m} \\ 100.00\,\boldsymbol{m} \end{bmatrix} \qquad \boldsymbol{x} = \begin{bmatrix} 99.97\,\boldsymbol{m} \\ 100.02\,\boldsymbol{m} \\ 141.44\,\boldsymbol{m} \\ 100.040^{g} \end{bmatrix}$$

$$\mathbf{l} = \boldsymbol{F}\left(\mathbf{X}^\circ\right) - \boldsymbol{x} = \begin{bmatrix} 0.030\,\boldsymbol{m} \\ -0.020\,\boldsymbol{m} \\ -0.020\,\boldsymbol{m} \\ -0.040^{g} \end{bmatrix}$$

Example

- Solution
- Solution with outlier

$$\hat{\mathbf{d}}_{\mathrm{x}} = \begin{bmatrix} -0.015\,m \\ -0.009\,m \\ 0.016\,m \\ -0.015\,m \\ -0.001\,m \\ 0.024\,m \end{bmatrix} \qquad \hat{\mathbf{v}} = \begin{bmatrix} \text{-}0.002\ m \\ \text{-}0.002\ m \\ 0.002\ m \\ \text{-}0.003^{g} \end{bmatrix}$$

$$\hat{\mathbf{d}}_{\mathrm{x}} = \begin{bmatrix} 0.317\,m \\ -0.184\,m \\ 0.851\,m \\ 0.317\,m \\ -1.168\,m \\ -0.133\,m \end{bmatrix} \qquad \hat{\mathbf{v}} = \begin{bmatrix} \text{-}0.002\ m \\ \text{-}0.002\ m \\ 0.002\ m \\ \text{-}0.002^{g} \end{bmatrix}$$

Example

- Solution

$$\hat{\mathbf{d}}_{\mathrm{x}} = \begin{bmatrix} -0.015\,m \\ -0.009\,m \\ 0.016\,m \\ -0.015\,m \\ -0.001\,m \\ 0.024\,m \end{bmatrix} \quad \hat{\mathbf{v}} = \begin{bmatrix} \text{-}0.002\ m \\ \text{-}0.002\ m \\ 0.002\ m \\ \text{-}0.003^{g} \end{bmatrix}$$

- Solution with outlier
  (with application
  of attenuation matrix)

$$\hat{\mathbf{d}}_{\mathrm{x}} = \begin{bmatrix} -0.003\,m \\ -0.026\,m \\ 0.051\,m \\ -0.003\,m \\ -1.967\,m \\ 0.026\,m \end{bmatrix} \quad \hat{\mathbf{v}} = \begin{bmatrix} \text{-}0.002\ m \\ \text{-}0.002\ m \\ 0.002\ m \\ \text{-}0.003^{g} \end{bmatrix}$$