KINEMATIC GPS BATCH PROCESSING, A SOURCE FOR LARGE SPARSE PROBLEMS

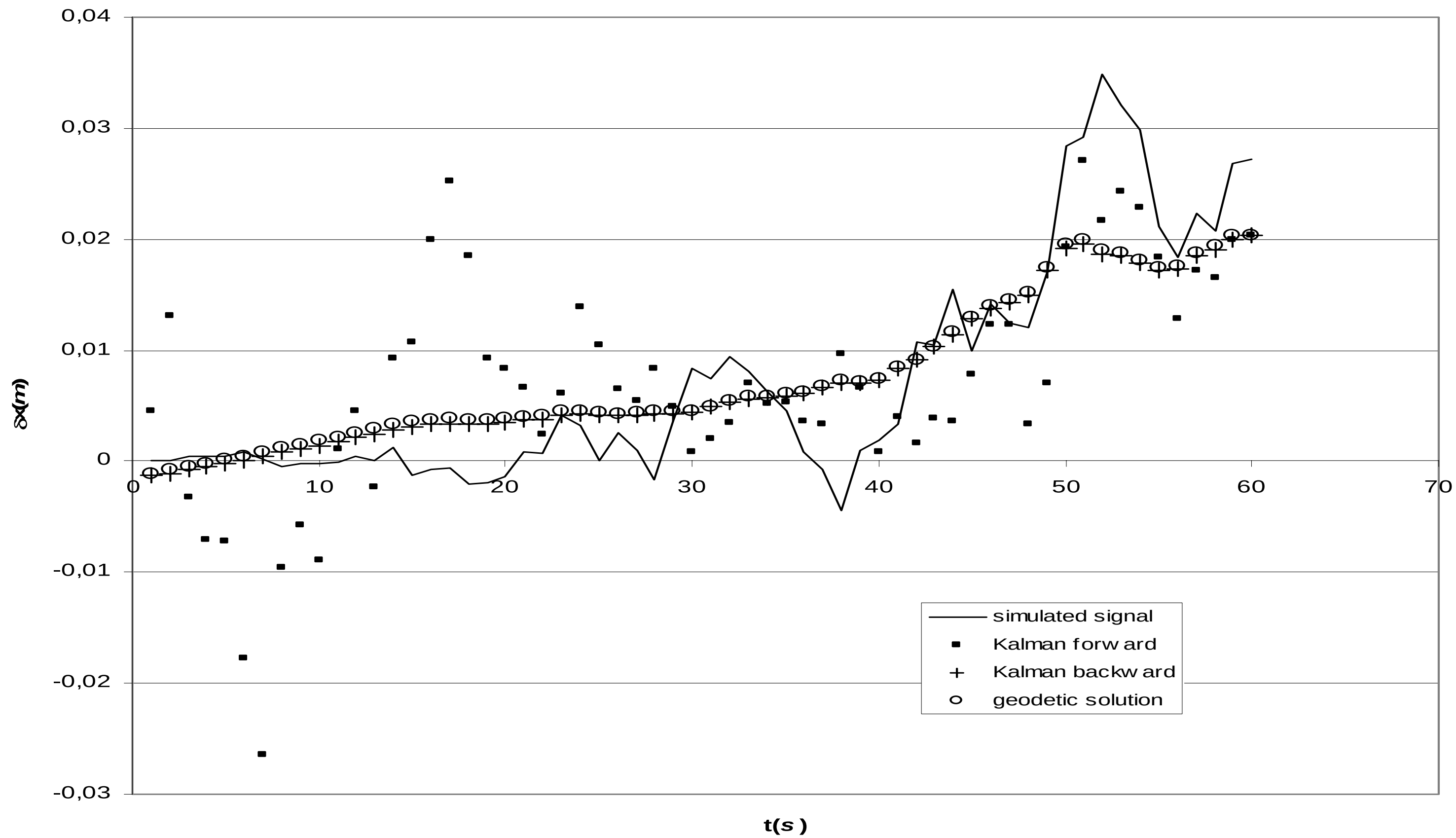
or

NOTES ON IMPROVING AMBIGUITY FIXING PERFORMANCES

Wroclaw - Poland
JUN, 22 – 24
2006

KINEMATIC GPS BATCH PROCESSING,
A SOURCE FOR LARGE SPARSE PROBLEMS

The equivalence between batch solution and state space approach has been shown (Sansò et al. 2006) and some advantages will be discussed.



Reference: Sansò et al., Real time and batch navigation solutions: alternative approaches (2006).

KINEMATIC DATA PROCESSING:
STATE SPACE VERSUS LMS NETWORK APPROACH

	STATE-SPACE APPROACH	NETWORK APPROACH
Real-Time parameter estimation	yes	no
Small matrices to be inverted	yes	no
Support for static observations	no	yes
Domain decomposition	no	yes

Reference: Colomina, Blàzquez, A unified approach to static and dynamic modeling in photogrammetry and remote sensing (2004)

CONSTANT BIAS ESTIMATIN IN KINEMATIC DATA

Consider a discrete-time linear system described by a finite state vector *x* and constant bias *b*, evolving with known dynamics:

$$x_{t+1} = T_{t+1} x_t + B_{t+1} b_t + \nu_{t+1}$$

$$y_{t+1} = H_{t+1} x_{t+1} + C_{t+1} b_{t+1} + \varepsilon_{t+1}$$

$$b_{t+1} = b_t$$

where the third equation shows that the bias vector is a constant, and the matrices *B* and *C* link the bias vector to system dynamic and to observations.

We introduce now some partitioned matrices

$$Z_t = \begin{array}{cc} \begin{array}{cc} p & n \end{array} & \\ \left[\begin{array}{c|c} T_t & B_t \\ \hline 0 & I \end{array}\right] & \begin{array}{c} p \\ n \end{array} \end{array} \qquad z_t = \left[\begin{array}{c} x_t \\ \hline b_t \end{array}\right] \begin{array}{c} p \\ n \end{array} \qquad G = \left[\begin{array}{c} I \\ \hline 0 \end{array}\right] \begin{array}{c} p \\ n \end{array} \qquad L_t = \begin{array}{cc} \begin{array}{cc} p & n \end{array} & \\ \left[\begin{array}{c|c} H_t & C_t \end{array}\right] & o \end{array}$$

The state and observation equation can now be expressed as

$$z_{t+1} = Z_{t+1} z_t + G \nu_{t+1}$$

$$y_{t+1} = L_{t+1} z_{t+1} + \varepsilon_{t+1}$$

The linear system has the optimal solution

M design matrix

W_ε weight matrix

$$\hat{z} = \left(\boxed{D^T W_\omega D} + \boxed{M^T W_\varepsilon M} \right)^{-1} M^T W_\varepsilon y = N^{-1} U$$

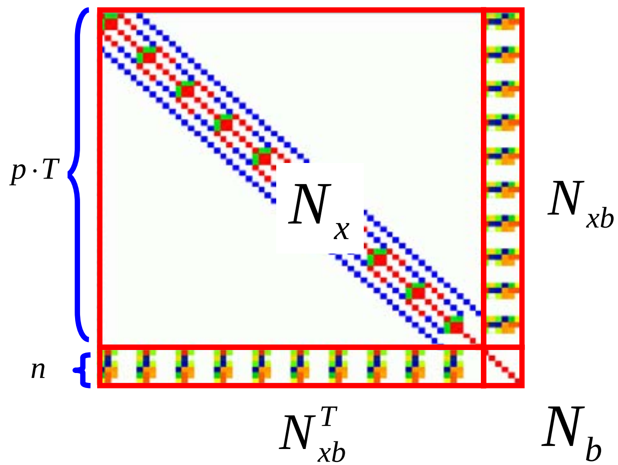
D dynamic matrix

W_ω weight matrix of dynamic

The matrix *D* and *E* are ordered and partitioned as follows

$$D = \begin{bmatrix} I & & & & -B_1 \\ -T_2 & I & & & -B_2 \\ & -T_3 & I & & -B_3 \\ & & \ddots & \ddots & \vdots \\ & & & -T_T \quad I & -B_T \end{bmatrix} \qquad M = \begin{bmatrix} H_1 & & & & C_1 \\ & H_2 & & & C_2 \\ & & \ddots & & \vdots \\ & & & H_T & C_T \end{bmatrix}$$

Fortunately, the large coefficient matrix to be inverted has a bordered block or band diagonal (BBD) structure, so the large matrix can thus be blockwisely inverted by using the following analytic inversion formula:



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} + A^{-1}BS^{-1}CA^{-1} & -A^{-1}BS^{-1} \\ -S^{-1}CA^{-1} & S^{-1} \end{bmatrix}$$

where *A*, *B*, *C* and *D* are matrix sub-blocks of arbitrary size, and *S* is the Schur complement

$$S = \left(D - CA^{-1}B\right) = N_b - N_{xb}^T N_x^{-1} N_{xb}$$

To estimate the constant bias vector, we can apply the Schur decomposition

$$\hat{b} = S_b^{-1}\left(u_b - N_{xb}^T N_x^{-1} u_x\right)$$

Note that we need only to invert N_x, that is large but is tridiagonal, and S_b that is small (has dimension equal to the number of constant biases).

Hypothesis:

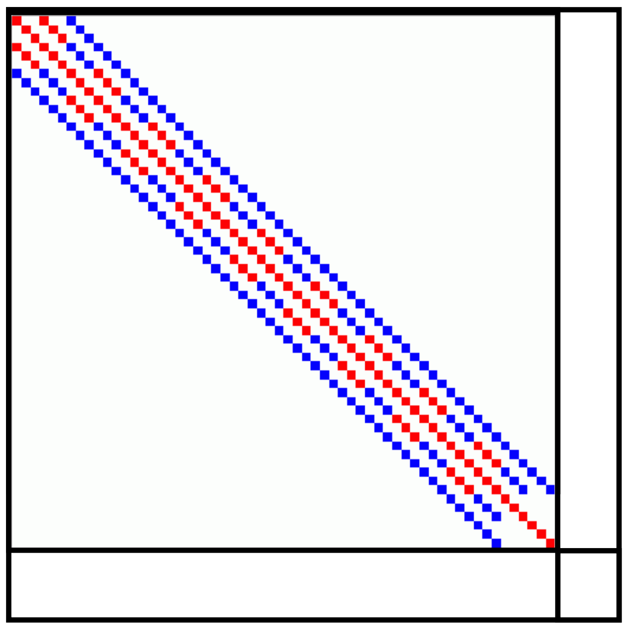
1. The bias vector has no direct connection to the parameters, but affect only the observations ($B = 0$).
2. The bias vector is constant or constant with steps, that is the case of carrier phase ambiguities affected by cycle slips; steps are taken in account by matrix C.
3. Matrix C, that link the bias vector to the observations, must be known a priori. So we need an algorithm to detect cycle slips before writing this matrix.

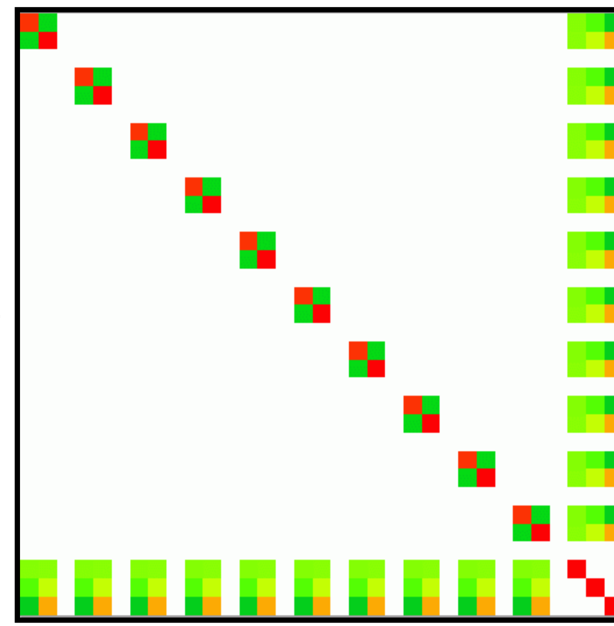
Expectation:

1. To smooth the estimated parameters according with a dynamic model.
2. To fix the integer ambiguities reducing the search area.

The case in wich $B = 0$ is of very practical interest. The normal matrix is now

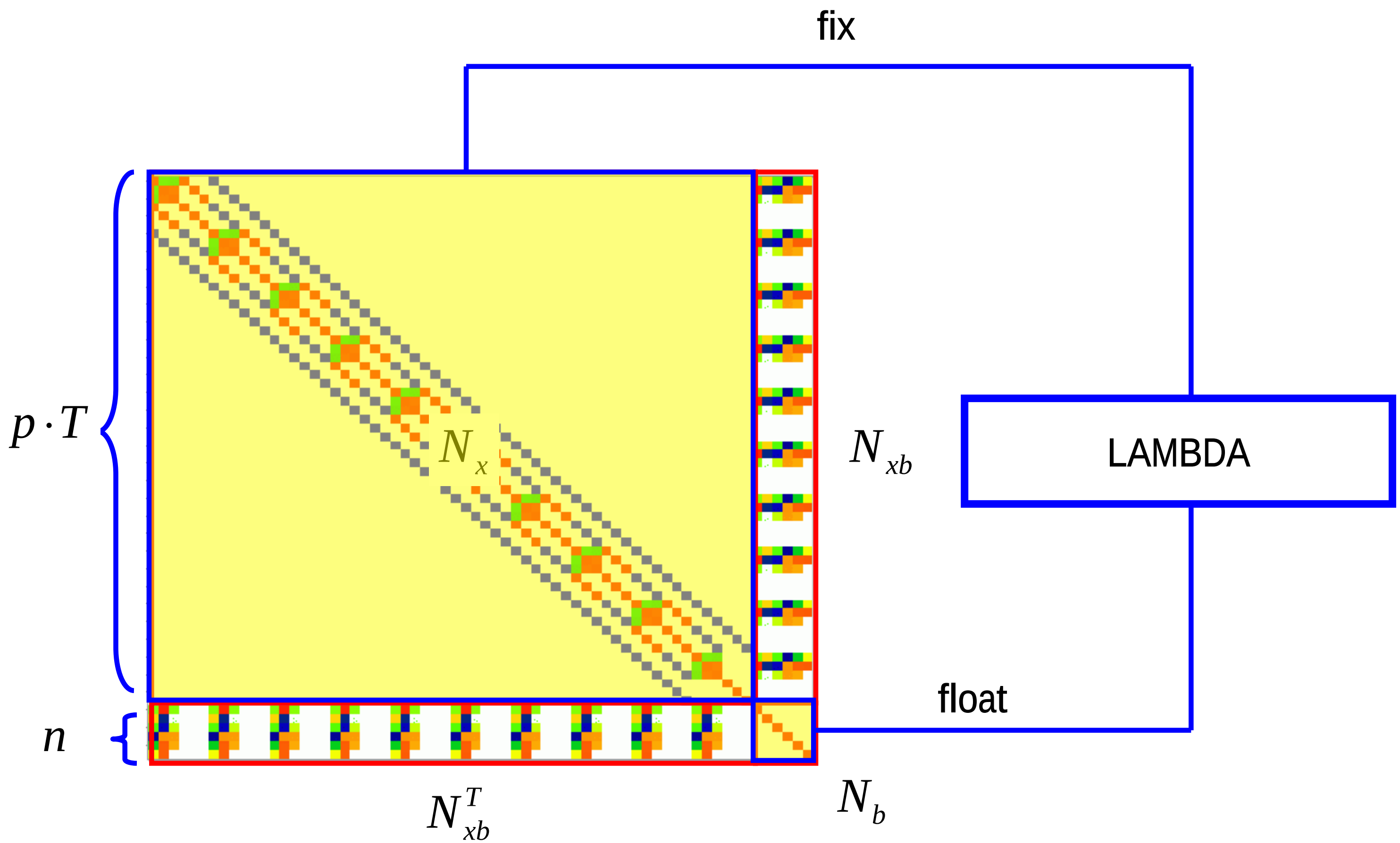
$$N = D^T W_\omega D + M^T W_\varepsilon M$$





The consequence is a reduction of the number of matrix sums and multiplications necessary to compute N. Matrix storage and inversion time are not changed.

ETIMATION OF INTEGER AMBIGUITIES



Wroclaw - Poland
JUN, 22 – 24
2006

1. Float ambiguities and variance-covariance matrix estimation.
2. Search for integer ambiguities (LAMBDA).
3. Correct double differences for ambiguities.
4. Solve for position inverting N_x.

LAMBDA is optimal in the sense of integer least squares, so why to use other methods?

LAMBDA performances will be improved acting on the float ambiguities and their variance-covariance matrix. Let the float solution be given as:

$$\begin{bmatrix} \hat{x} \\ \hat{b} \end{bmatrix}, \begin{bmatrix} Q_{\hat{x}} & Q_{\hat{b}\hat{x}} \\ Q_{\hat{x}\hat{b}} & Q_{\hat{b}} \end{bmatrix}$$

The decorrelation matrix Z^T transforms the ambiguities and their variance-covariance matrix into

$$\hat{z} = Z^T \hat{b}, \quad Q_{\hat{z}} = Z^T Q_{\hat{b}} Z$$

The elements of matrix Z are all integers and in order to be volume preserving its determinant must be ±1.

The minimization problem of integer least squares estimation is

$$\min \left(\hat{z} - z \right)^T Q_{\hat{z}}^{-1} \left(\hat{z} - z \right), \quad z \in Z^n$$

The integer values of the original ambiguities are obtained by the inverse of transformation

$$\breve{b} = Z^{-T} \breve{z}$$

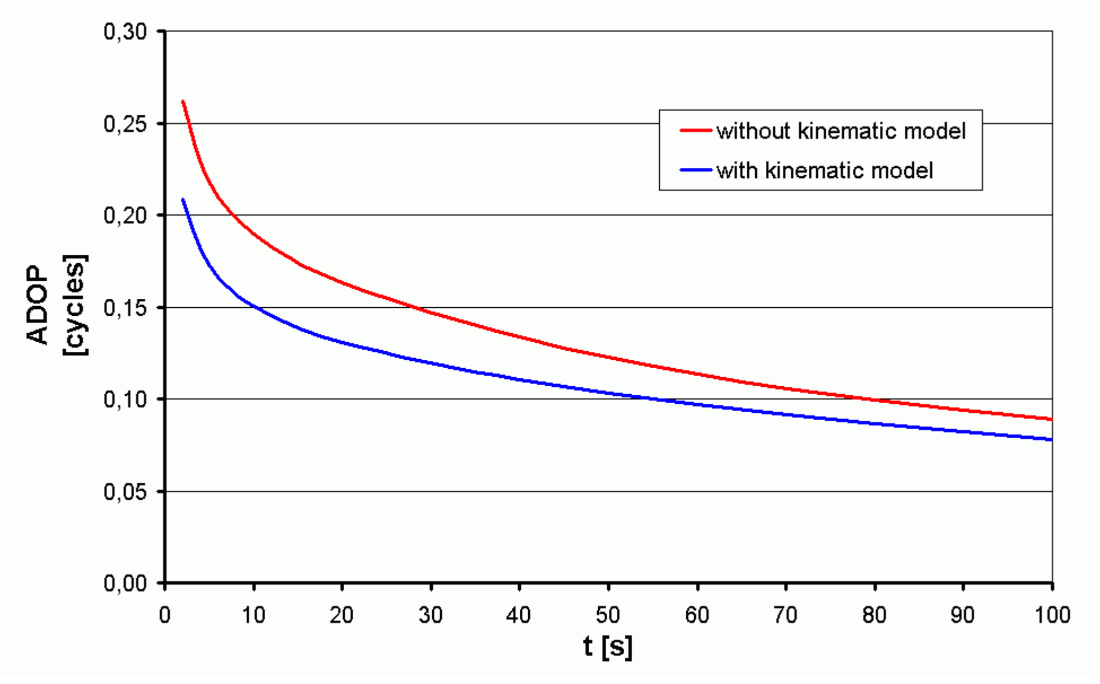
Reference: P. Teunissen et al., various pubblications, Delft.

ADOP

The ambiguity dilution of precision is a uniquely defined scalar reflecting the accuracy of float ambiguities

$$ADOP = \sqrt{\left|Q_{\hat{b}}\right|}^{\frac{1}{n}}$$

It is invariant to a class of ambiguity transformations (choice of the reference satellite, decorrelation of LAMBDA, ecc.). It depends on observation accuracy and redundancy.



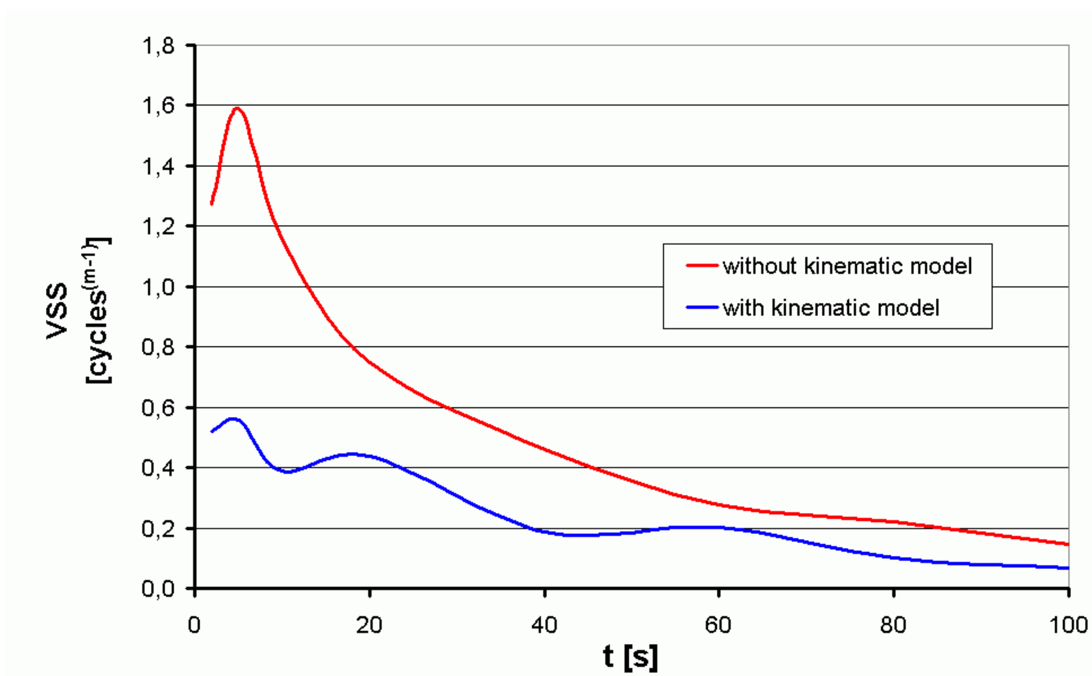
Wroclaw - Poland
JUN, 22 – 24
2006

VOLUME OF THE SEARCH SPACE

The volume of the ambiguity search space is given as

$$V_n = \chi^{2n} \boxed{U_n} \sqrt{\left|Q_{\hat{b}}\right|}$$

Volume of the unit sphere in R^n

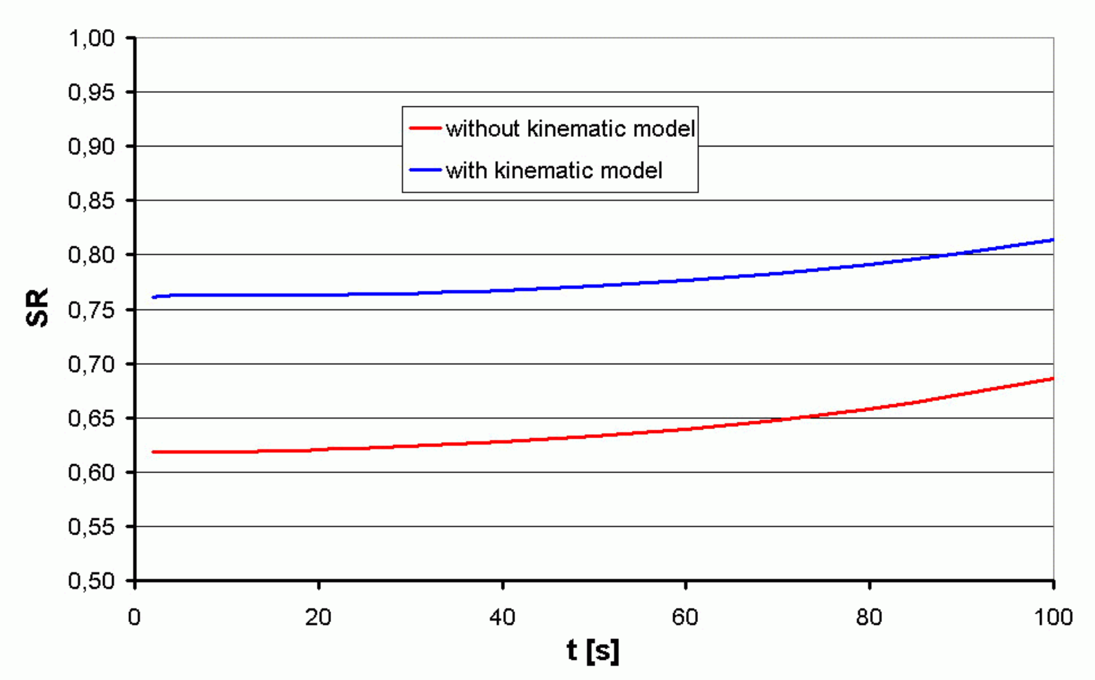


SUCCESS RATE

The lower bound of success rate is given by

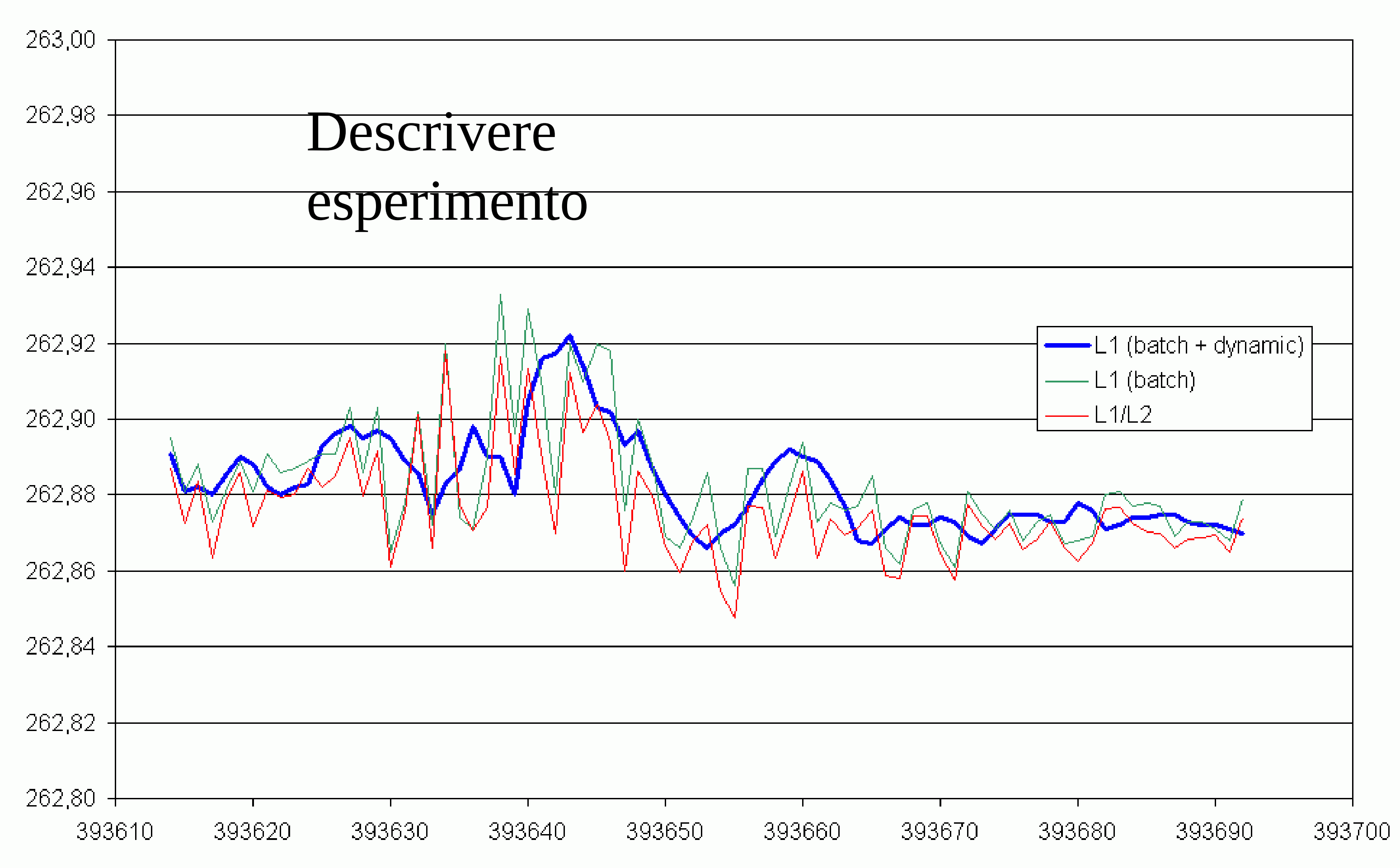
$$P\left(\breve{z}=z\right)\geq\prod_{i=1}^{n}\left(2\Phi\left(\frac{1}{2\sigma_{\hat{z}_{i|I}}}\right)-1\right)$$

where n is the number of ambiguities and $\Phi(x)$ is the probability mass function; the standard deviation σ is the square root of the conditional variance of the i^{th} ambiguity.



CONCLUSIONS

Double frequency solutions by commercial software have been compared with the single frequency batch solution. Observation taken at 1” data rate.



CONCLUSIONS

- Constraining dynamic in Least Square estimation of float ambiguities:
  - the success rate for LAMBDA method have been increased, reducing ADOP and the volume of search space;
  - the robustness of cycle slip fixing have been increased;
  - the effect of noise on the estimated trajectory have been reduced, according to the kinematic model.
- The computational load is comparable to Kalman filtering + smoothing, thanks to the structure of the normal matrix and of domain decomposition.

OPEN QUESTIONS

- The effect of the choice of the (deterministic and stocastic) kinematic model is under investigation.
- A priori hypothesis on the stocastic model of the observation.